A Solution to the Forward Kinematic Problem of the ARma6 Robot

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***Abstract*—The use of robots is expanding in industrial, medical, educational, and research domains, where they play a crucial role in automation and advanced technological applications. Forward kinematics is a fundamental component of robot design, control, and simulation. This paper presents the solution for forward kinematics problem for the custom-designed 6DoF ARma6 robot, including the assignment of joint frames, determination of Denavit–Hartenberg parameters, and computation of homogeneous transformation matrices to obtain the Tool Centre Point pose. Arma6 is an articulated robot that follows a UR robot-type kinematic structure, and is intended as a lightweight and cost-effective platform for research applications. The forward kinematics algorithm provides the foundation for the solution of the inverse problem, trajectory planning, motion control, and robotic simulation.**

***Index Terms*—robot, forward kinematics, Denavit-Hartenberg, homogeneous transformation.**

I. INTRODUCTION

The articulated robot, sometimes also called a jointed, elbow, or anthropomorphic manipulator, is widely used in industry, medicine, research, and other domains [1], [2]. Robot kinematics is a fundamental aspect of development, analysis, control and simulation in robotics [3]. An articulated robot is modeled as an open kinematic chain with multiple degrees of freedom [2]. In order for a robotic manipulator to move in a three-dimensional space where it performs certain tasks, it is necessary to control the position and orientation (pose) of the end-effector, i.e. its Tool Centre Point (TCP).

The task of forward (direct) kinematics refers to determining the position and orientation of the robot's end-effector for the input values of the robot's joints positions [4]. The solution of the direct robot kinematic problem is used in several important segments of robot control: As a basis for the inverse kinematics algorithm; for planning the trajectory, for analyzing the workspace, for avoiding collisions, etc.

ARma6 (Fig. 1) is a custom-designed, cost-effective articulated 6-DOF robot currently under development, featuring an open-architecture controller and intended for research applications involving XR-based robot programming systems [5]. The kinematic structure of ARma6 is typical of UR-type robots [6], which feature a distinctive serial kinematic structure with parallel joint 2–4 axes, differing from conventional shoulder–elbow configurations, and have the motor, and harmonic drive gearing for each joint directly at the joint (enabling a compact and cost-effective design). One of the main kinematic characteristic of UR type robots is that the last three joints do not act as a coincidental wrist (the spherical joint is not formed) [7]. Therefore all its six joints contribute to the transformational and rotational movements of its end-effector [7].

Fig. 1: ARma6 robot prototype.

In this paper, forward kinematics algorithm for ARma6 robot is presented. The methodology for the definition of joint frames is given, and the frame transformations and determination of the Denavit-Hartenberg (DH) parameters [8] are described. The homogeneous transformation matrices (HTM) are consequently calculated, and the TCP pose is determined.

II. FRAME ASSIGNMENT AND DEFINITION OF DENAVIT-HARTENBERG PARAMETERS

One of the most used methods for kinematic modeling in robotics is the Denavit–Hartenberg convention [8] which is adopted in this paper.

A. *Joint frame assignment*

Using the convention for robot joints' frames definition applied in this study, the axis z_i is assigned as the axis of actuation of joint $i+1$ [9]. The following rules for joint frame assignment have been adopted: The x_i axis intersects the z_{i-1} axis and is perpendicular to it [9]. The assigned frames of ARma6 robot are presented in Fig.2. The BASE frame coincides with the zero (0) frame herein.

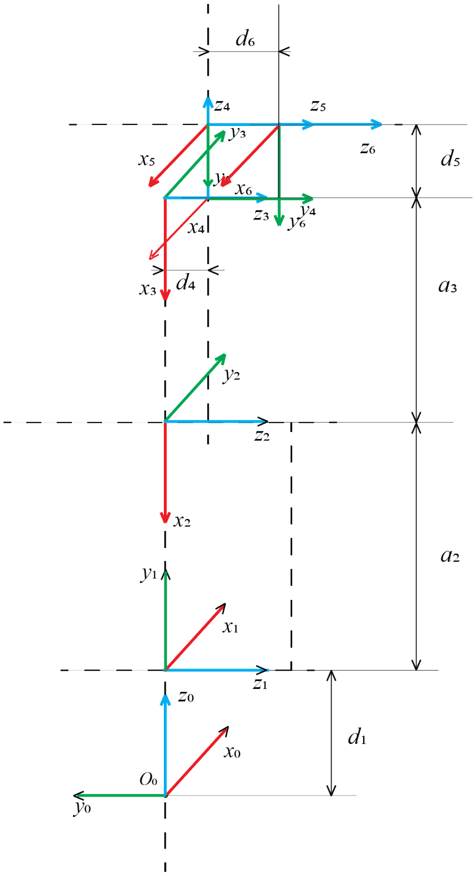


Fig. 2: Frames assigned to ARma6 joints.

B. *Table of Denavit-Hartenberg parameters*

The 4×4 matrix known as HTM maps a homogeneous position vector from one coordinate system to another. If several transformations (rotations or translations) are performed in succession, the HTM describing the overall transformation is obtained by multiplying HTMs describing the successive individual transformations. The HTM describing coordinate frame i relative to frame $i-1$ using Denavit–Hartenberg parameters has the form:

$$\mathbf{A}_i = \mathrm{Rot}(z,\theta_i)\,\mathrm{Trans}(z,d_i)\,\mathrm{Trans}(x,a_i)\,\mathrm{Rot}(x,\alpha_i) \qquad (1)$$

In this study, DH parameters, Table 1, have been determined based on transformation of frames attached to links, Fig.3.

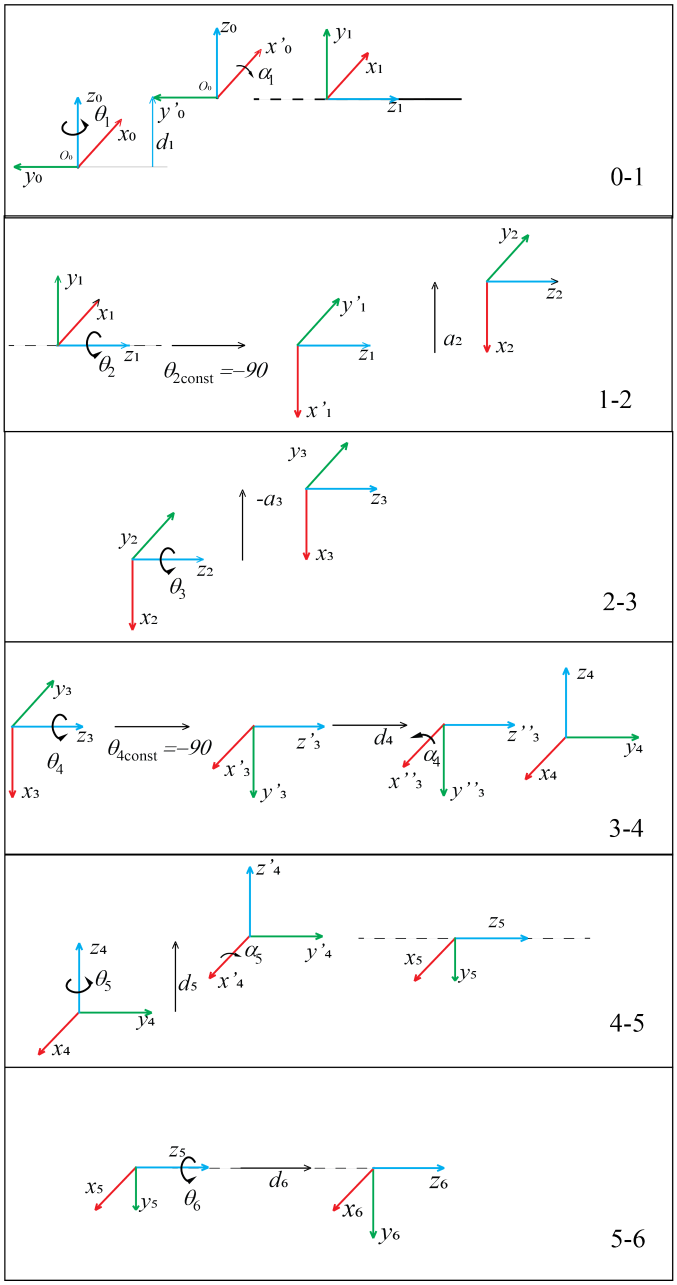


Fig. 3: Frames transformations

TABLE I: DH parameters

Joint i	a_i	α_i	d_i	θ_i
1	0	90°	d_1	θ_1
2	a_2	0	0	$\theta_2-90°$
3	a_3	0	0	θ_3
4	0	90°	d_4	$\theta_4-90°$
5	0	-90°	d_5	θ_5
6	0	0	d_6	θ_6

III. CALCULATION OF HOMOGENEOUS TRANSFORMATION MATRICES AND TCP POSE DETERMINATION

A. Joint-to-joint homogeneous transformation matrices

In the following, HTMs for joints $i = 1$ to 6 are presented. From this point, c_i and s_i denote the cosine and sine of the rotation angle θ_i about the z_{i-1} axis, i.e., the i-th internal coordinate q_i of the robotic manipulator.

$$\mathbf{A}_1 = \mathrm{Rot}_z(q_1)\ \mathrm{Trans}_z(d_1)\ \mathrm{Rot}_x\left(\frac{\pi}{2}\right) \tag{2}$$

which yields:

$$\mathbf{A}_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\mathbf{A}_2 = \mathrm{Rot}_z(q_2)\ \mathrm{Rot}_z\left(-\frac{\pi}{2}\right)\ \mathrm{Trans}_x(-a_2) \tag{4}$$

which yields:

$$\mathbf{A}_2 = \begin{bmatrix} s_2 & c_2 & 0 & -a_2 s_2 \\ -c_2 & s_2 & 0 & a_2 c_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$\mathbf{A}_3 = \mathrm{Rot}_z(q_3)\ \mathrm{Trans}_x(-a_3) \tag{6}$$

which yields:

$$\mathbf{A}_3 = \begin{bmatrix} c_3 & -s_3 & 0 & -a_3 c_3 \\ s_3 & c_3 & 0 & -a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$\mathbf{A}_4 = \mathrm{Rot}_z(q_4)\ \mathrm{Rot}_z\left(-\frac{\pi}{2}\right)\ \mathrm{Trans}_z(d_4)\ \mathrm{Rot}_x\left(\frac{\pi}{2}\right) \tag{8}$$

which yields:

$$\mathbf{A}_4 = \begin{bmatrix} s_4 & 0 & -c_4 & 0 \\ -c_4 & 0 & -s_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$\mathbf{A}_5 = \mathrm{Rot}_z(q_5)\ \mathrm{Trans}_z(d_5)\ \mathrm{Rot}_x\left(-\frac{\pi}{2}\right) \tag{10}$$

which yields:

$$\mathbf{A}_5 = \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$\mathbf{A}_6 = \mathrm{Rot}_z(q_6)\ \mathrm{Trans}_z(d_6) = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

B. Calculation of the position and orientation of the frame attached to the TCP

Based on joint-to-joint HTMs (2-12), the HTM of the frame attached to the robot's TCP with the respect to BASE frame is obtained as:

$$\mathbf{T}_{06} = \mathbf{A}_1\ \mathbf{A}_2\ \mathbf{A}_3\ \mathbf{A}_4\ \mathbf{A}_5\ \mathbf{A}_6 \tag{13}$$

By adopting the following substitutions:

$$v_1 = c_1 c_5, v_2 = s_1 s_5, v_3 = c_{234} v_1 = c_{234} c_1 c_5, v_4 = s_{234} c_1, \tag{14}$$

$$v_5 = c_1 s_5 + c_{234} v_2, v_6 = d_6 c_5 + d_4, \tag{15}$$

$$v_7 = d_6 c_{234} s_5 - d_5 s_{234} - a_2 s_2 - a_3 \sin(q_2 + q_3) \tag{16}$$

$$v_8 = d_1 + a_3 c_{23} + a_2 \cos(q_2) + d_5 c_{234} + d_6 \sin(q_5) s_{234} \tag{17}$$

$$c_{234} = \cos(q_2 + q_3 + q_4), s_{234} = \sin(q_2 + q_3 + q_4) \tag{18}$$

the HTM of the TCP frame with respect to the BASE frame is obtained, and it is given in (19).

When the HMT $\mathbf{T}_{06}$ is determined, the position and orientation of the TCP with respect to the BASE frame can be calculated for every time instance given that $\mathbf{T}_{06}$ contains the rotation matrix of the TCP frame wrt. the BASE frame and the position vector of the TCP frame's origin wrt. the BASE frame. The calculated HTM $\mathbf{T}_{06}$ can be presented as:

$$\mathbf{T}_{06} = \begin{bmatrix} n_x & o_x & a_x & X \\ n_y & o_y & a_y & Y \\ n_z & o_z & a_z & Z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where X, Y, Z are the coordinates of the origin of the TCP frame wrt. BASE frame, and $(n_x, n_y, n_z), (o_x, o_y, o_z), (a_x, a_y, a_z)$ define the TCP frame axes wrt. the BASE frame.

The Euler angles are calculated from the rotation matrixbased on the adopted convention. Herein, the ORIZYX convention (Rot(z, A)Rot(y', B)Rot(x'', C)) is used, and Euler angles are computed as [10]:

$$A = \arctan 2(n_y, n_x), \tag{20}$$

$$B = \arctan 2(-n_z, n_x \cos A + n_y \sin A), \tag{21}$$

$$C = \arctan 2(a_x \sin A - a_y \cos A, -o_x \sin A + o_y \cos A), \tag{22}$$

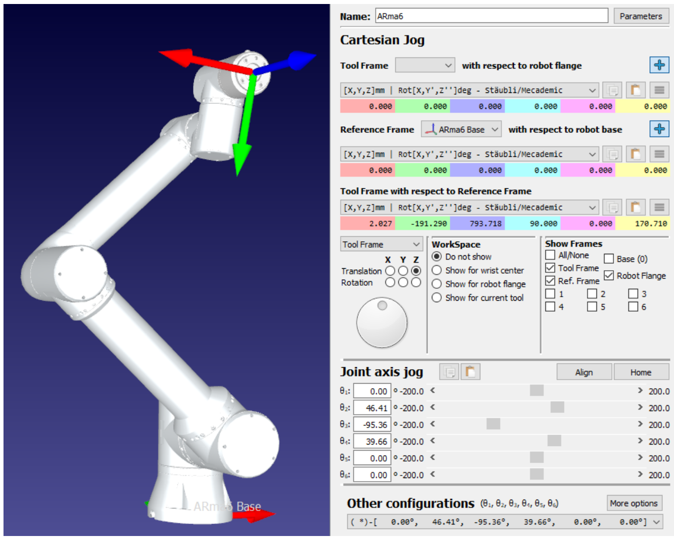
where:

- A is the rotation around the base Z-axis (yaw),
- B is the rotation around the rotated y'-axis (pitch),
- C is the rotation around the twice-rotated x''-axis (roll).

$$\mathbf{T}_{06} = \begin{bmatrix} c_6(v_2 - v_3) + s_6 v_4 & s_6(v_2 - v_3) - c_6 v_4 & v_1 + s_1 c_{234} s_5 & s_1 v_7 - c_1 v_6 \\ s_{234} s_1 s_6 - c_6(c_1 s_5 + v_3) & s_6(c_1 s_5 + v_3) + s_{234} c_6 s_1 & c_{234} s_1 s_5 - c_1 c_5 & -c_1 v_7 - s_1 v_6 \\ -c_{234} s_6 - s_{234} c_5 c_6 & s_{234} c_5 s_6 - c_{234} c_6 & s_{234} s_5 & v_8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

C. *Forward Kinematics Algorithm Validation in RoboDK*

For verification purposes, Arma6 was modeled in RoboDK software [11] using the "Model Mechanism or Robot" tool. BASE frame and TCP frame are positioned and oriented in RoboDK simulator to coincide with frames given in Fig. 2. The developed forward kinematics algorithm was validated using various robot configurations, and the results of the TCP pose matched the RoboDK outputs, confirming the algorithm. Fig. 4 shows a screenshot of the RoboDK software displaying the ARma6 robot model, used for validating the forward kinematics algorithm.



Name: ARma6

Cartesian Jog

Tool Frame with respect to robot flange

[X,Y,Z]mm | Rot[X,Y',Z'']deg - Stäubli/Mecademic

0.000	0.000	0.000	0.000	0.000	0.000

Reference Frame ARma6 Base **with respect to robot base**

[X,Y,Z]mm | Rot[X,Y',Z'']deg - Stäubli/Mecademic

0.000	0.000	0.000	0.000	0.000	0.000

Tool Frame with respect to Reference Frame

[X,Y,Z]mm | Rot[X,Y',Z'']deg - Stäubli/Mecademic

2.027	-191.290	793.718	90.000	0.000	170.710

Tool Frame

	X	Y	Z
Translation	○	○	●
Rotation	○	○	○

WorkSpace
● Do not show
○ Show for wrist center
○ Show for robot flange
○ Show for current tool

Show Frames
☐ All/None
☑ Tool Frame
☑ Ref. Frame
☐ Base (0)
☑ Robot Flange
☐ 1 ☐ 2 ☐ 3
☐ 4 ☐ 5 ☐ 6

Joint axis jog

θ₁: 0.00 ° -200.0 200.0
θ₂: 46.41 ° -200.0 200.0
θ₃: -95.36 ° -200.0 200.0
θ₄: 39.66 ° -200.0 200.0
θ₅: 0.00 ° -200.0 200.0
θ₆: 0.00 ° -200.0 200.0

Other configurations (θ₁, θ₂, θ₃, θ₄, θ₅, θ₆)

(*)-[0.00°, 46.41°, -95.36°, 39.66°, 0.00°, 0.00°]

Fig. 4: Forward Kinematics Algorithm Validation in RoboDK

IV. CONCLUSION

In this paper, the forward kinematics of the ARma6 robot, an articulated 6-DoF manipulator following a UR-type kinematic structure, was formulated using Denavit–Hartenberg parameters and homogeneous transformation matrices. The model determines the tool centre point pose and provides the basis for inverse kinematics, trajectory planning, and motion control. The presented forward kinematics algorithm is verified through simulation using the complete robot model in RoboDK software.

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