

# Pricing American Options using Artificial Neural Networks and Monte Carlo Method

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**Abstract**— Pricing American-style options is a challenging problem due to the flexibility of early exercise. Closed-form analytical solutions are generally unavailable, particularly for models with complex dynamics. This paper presents a detailed exposition of the Neural Network Monte Carlo (NNMC) method for pricing American options. The method approximates continuation values via regression on suitable artificial neural network model. We provide a step-by-step description of the algorithm, numerical examples, and a comparative analysis with binomial models, highlighting convergence properties and the early exercise values of the options.

**Keywords**—American option, Monte Carlo simulation, neural networks, regression

## I. INTRODUCTION (HEADING 1)

American options grant the holder the right to exercise at any time prior to maturity, distinguishing them from European options and introducing a nontrivial optimal stopping problem. This flexibility significantly complicates their valuation, as the holder must decide at each time whether immediate exercise or continuation maximizes expected payoff. Consequently, American option pricing has attracted considerable attention in both financial engineering and computational science.

Early numerical approaches relied on lattice-based techniques, most notably the binomial and trinomial tree models introduced by Cox, Ross, and Rubinstein [1]. These methods discretize both time and the underlying asset price and approximate the optimal exercise boundary via backward induction. Finite-difference methods were later developed to solve the free-boundary partial differential equations associated with American options [2]. While effective for low-dimensional problems, such approaches suffer from the curse of dimensionality and become impractical when multiple state variables are involved.

Monte Carlo simulation is widely used in computational finance due to its flexibility and favorable scaling in high dimensions. However, standard Monte Carlo methods are inherently forward-looking and therefore unsuitable for problems involving early exercise. A major breakthrough was achieved by Longstaff and Schwartz [3], who proposed the Least-Squares Monte Carlo (LSMC) method. Their approach approximates continuation values using regression on

simulated paths, enabling efficient valuation of American-style derivatives even in complex settings.

Subsequent research has focused on improving the accuracy and convergence properties of the LSMC method through the choice of basis functions, variance reduction techniques, and alternative regression schemes [4], [5].

Recent advances in American option pricing build upon the classical Least-Squares Monte Carlo (LSMC) framework by incorporating modern regression techniques and machine-learning methodologies. Gaussian process regression combined with deep kernel learning has been shown to significantly improve pricing accuracy in high-dimensional settings, particularly under models with jumps and stochastic volatility [6]. Neural-network-based approaches, including Kolmogorov-Arnold Networks, replace predefined basis functions with data-driven representations, yielding more robust continuation value estimates and improved data efficiency [7].

Further methodological enhancements include gradient-enhanced sparse polynomial expansions, which improve both pricing accuracy and sensitivity estimation (Greeks) while mitigating the curse of dimensionality [8]. From a computational perspective, distributed and parallel implementations of LSMC have been proposed to scale Monte Carlo simulations efficiently across large computing infrastructures, substantially reducing runtime for large-scale problems [9]. Additional variance-reduction strategies, such as importance sampling combined with shifted regression schemes, have demonstrated meaningful reductions in estimator bias and variance [10], while generic regression improvements for optimal stopping problems further strengthen the theoretical foundation of modern LSMC techniques [11].

This paper extends the standard exposition of the LSMC framework by introducing the regression model based on artificial neural networks. Instead of using standard regression models for the prediction of the continuation value, we implemented the neural networks for this purpose. The paper presents a detailed algorithmic description, numerical illustrations, and an extensive comparison with the binomial and LSMC pricing methods.

## II. PRICING AMERICAN OPTIONS

### A. Overview

An American option gives the right to be exercised at any time before maturity. Under the risk-neutral measure, its value is defined as the supremum over all stopping times of the expected discounted payoff, where the values of a call and a put option are given respectively as

$$C = \max_{\tau} E \left[ e^{-\int_0^{\tau} r(w,s) ds} \max \{ S(\tau) - K, 0 \} \right],$$

$$P = \max_{\tau} E \left[ e^{-\int_0^{\tau} r(w,s) ds} \max \{ K - S(\tau), 0 \} \right],$$

for all the moments  $\tau \leq T$ .  $r(w, s)$  is the possible risk-free interest, related to given tendency of development  $w$ . For simplicity we will assume that risk-free rate is constant, i.e. doesn't depend on  $w$ . Further,  $K$  is the strike price, defined through the contract agreement, while  $S(\tau)$  is the price of the underlying security at moment  $\tau$ .

Unlike European options, valuation requires determining an optimal exercise boundary separating exercise and continuation regions. At each exercise date, one has to decide whether to exercise the option or to wait. In order to price an American option we must determine *the optimal exercise boundary* i.e. the boundary which separates the early exercise and the continuation regions.

Some analytical solutions for American options have been proposed, but there is no easily computable formulae currently available, and again numerical methods are required. Also, it may happen that very strict assumptions implemented in analytical solutions can lead to an (empirically) unrealistic model. Above all, when the assets have some complex features such as multiple stochastic processes, non-Markov property etc. and they cannot be priced, in general case, by any analytical method.

Due to the lack of general closed-form solutions, numerical methods are required. These include lattice methods, finite-difference schemes, quadrature methods, and Monte Carlo simulation. Monte Carlo methods simulate paths of the underlying security. This approach is particularly attractive when dealing with complex dynamics or multiple state variables.

In the paper we are dealing with one underlying asset. We simulate evolution of the value of this asset over the time interval  $[0, T]$ , where  $T$  is time to maturity. This interval is discretized into  $m$  mutually equal subintervals. The general assumption is that the price of the underlying asset  $S(t)$  follows the stochastic differential equation

$$dS(t) = rS(t)dt + \sigma S(t)dW(t),$$

where  $r$  is a growth rate (in our case it is a constant risk-free rate),  $\sigma$  is a volatility of the process  $S(t)$  and  $W(t)$  is the Brownian motion.

In order to simulate the dynamic of the above differential equation we will use the Euler approximation  $S_i$  of  $S(t_i)$  on a time grid  $0 = t_0 < t_1 < \dots < t_m = T$ . It is defined as

$$S_{i+1} = S_i + r S_i [t_{i+1} - t_i] + \sigma S_i \sqrt{t_{i+1} - t_i} Z_{i+1}.$$

The initial value  $S_0$  is the current market price of the underlying asset. The random variables  $Z_i$  are independent identically distributed, following the standard normal distribution.

### B. Neural Network Approach

The core idea of the NNMC method is to approximate the continuation value of the option using artificial neural networks. Time is discretized into a finite number of exercise dates, and a large number of asset price paths are simulated under the risk-neutral measure.

We assume that there exist only  $m$  exercise dates,  $0 = t_0 < t_1 < \dots < t_m = T$ . For a sufficiently large number  $m$  we converge to the real value of American option (which can be exercised continuously in time). The continuation value is equal, under no-arbitrage condition, to the risk-neutral expectation of the discounted future cash flow, i.e.

$$V(t_i) = E^Q \left[ \sum_{j=i+1}^m e^{-\int_{t_i}^{t_j} r(s) ds} C(t_j; t_i, T) \vee F_{t_i} \right],$$

where  $F_{t_i}$  is the information set up to time  $t_i$ .  $C(t_j; t_i, T)$  are future cash flows that can occur in the future, up to maturity. The idea is that this conditional expectation can be approximated using the neural network approach.

At each exercise date, the continuation value is estimated by regressing discounted future cash flows on a set of previous underlying asset values. The regression model is a single hidden layer feedforward neural network defined as

$$\sum_{i=1}^L \alpha_i h(w_i \cdot x_j + b_i) = f_j,$$

where  $w_i = [w_{i1}, w_{i2}, \dots, w_{iL}]^T$  is a weight vector that connects the  $i^{\text{th}}$  hidden neuron and all input neurons. Vector  $\alpha_i = [\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{i3}]^T$  is a vector of weights that connects  $i^{\text{th}}$  hidden neuron with the output neuron, and  $b_i$  is a bias of the  $i^{\text{th}}$  neuron.

The input layer has three neurons and they contain values of the underlying asset from the previous three moments. The hidden layer has 15 neurons, while the output layer will the value of the discounted payoff.

Parameter estimation of this model is conducted by the back-propagation algorithm, where the initial values for weights and bias are set arbitrarily.

### C. Neural Network Monte Carlo Method

The first step of this method is to generate enough number of paths that represent the evolution of the underlying asset price up to maturity  $T$ . For that purpose generate set of 1,000 sample paths. For these paths the risk-free rate and the volatility are set to be constant.

Starting for the moment  $t_m = T$  we first calculate payoff for each sample path. The question that arise here is whether the payoff would be bigger if the option was exercised at the moment  $t_{m-1}$ . So, the payoff at  $t_m$  is discounted to the moment

$t_{m-1}$ . That value will be used to train the neural network. For the given input vector, the output will be the continuation value. The exercise decision is then made by comparing the immediate payoff with the estimated continuation value.

Thus, at each moment  $t_2 < t_k < t_m$ , we will need the trained neural network whose output value is the continuation value for the moment  $t_k$  for a particular sample path. The process goes back in time, only for the paths that are in-the money. Namely, for some moment  $t_k$  if process is out-of-the money, immediate payoff is zero, thus we will always consider the continuation value as the favorable choice. Accordingly, in order to train the neural network we will use only in-the money sample paths for the moment  $t_k$ . Then, in order to make the exercise decision, the neural network estimates the continuation value for the in-the money paths. If the output value is bigger than the discounted future payoff, the decision is to exercise the option at  $t_k$ . The payoff generated this way (after discounting for moment in time) will be used to train a new network for the moment  $t_{k-1}$ .

This backward induction continues until the initial time, i.e. until  $t_3$ , since the first three values are necessary to train the model. For sufficiently large  $m$ , it can be assumed that the option will not be exercised in the first three moments. Finally, across all sample paths, we will observe and collect the payoffs, and discount these values to the moment  $t_0$ . Average of the discounted payoff values will be the value of the American option. For all discounting mentioned in the previous discussion, the same rate  $r$  is used that figures through the entire model.

### III. NUMERICAL RESULTS

In this section we will discuss the results obtained with the NNMC method when using different discretization of the holding period. These results will be compared with the results obtained using the binomial method [1] and the LSMC method [3]. Also, for the comparison reason the value of the Black-Scholes model will be given [4]. Although it provides values for European options (where no early exercise is possible), we will use the Black-Scholes model to observe the early exercise values for a call and put options of the American type.

We set the time to maturity to be one year, where we discretize this period to time interval  $\Delta t$ , where

$$\Delta t \in \{0.05, 0.0125, 0.004\}.$$

Corresponding number of steps for the binomial method will be 20, 80 and 250 steps respectively. The options we are going to price are an American call and an American put option with spot price  $S_0=100$ , time to maturity  $T=1$ , risk-free rate  $r=0.05$ , volatility  $\sigma \in \{0.3, 0.5\}$  and strike prices  $K \in \{90, 100, 110\}$ . The results are summarized in Table I and Table II.

Table I and Table II also report early exercise values. The early exercise value is a difference between the American and European option values, where the European call and put prices are given by the Black-Scholes equation.

The early exercise component is the most difficult component to determine when pricing American style options. From a theoretical perspective, it is well known that an American call option written on a non-dividend-paying underlying asset should never be exercised prior to maturity [4]. Consequently, the value of such an American option coincides with, or is extremely close to, the value of the corresponding European option. Although the American option price is always at least as large as the European price, the early exercise premium in this setting is effectively zero. The numerical results presented in Table I confirm this theoretical property, as the all three estimates closely match the corresponding European benchmark values.

Further, the results reported in Table I indicate that the Binomial and LSMC method are more sensitive to the holding period discretization. When  $\Delta t=0.0125$ , the methods produces reasonably accurate estimates, however, systematic underestimation of the option value is observed under certain parameter configurations. In contrast, NNMC provides more robust results, where only for  $\Delta t=0.08$  it shows some deviations. Nevertheless, even with the NNMC method, sufficiently fine time discretization remains necessary to ensure reliable convergence.

The binomial method, while conceptually straightforward and widely used, also tends to underestimate option values when the number of time steps is limited. Accurate approximation of the option price therefore requires a very fine time grid, which increases computational cost.

In the second part of the analysis, American put options are considered under the same market parameters as the previously examined call options. The corresponding results are summarized in Table II, with binomial estimates again serving as a benchmark for comparison. Unlike the call case, early exercise plays a significant role in the valuation of American put options. Table II highlights that the early exercise premium constitutes a substantial component of the total option value. The graphical representation of this discussion is presented by Figure I. For practical reason only the early exercise values from Table II are given. We can notice that, in most cases, the early exercise values are more stable with NNMC than with other two methods.

Furthermore, the numerical evidence suggests that the NNMC method converges more rapidly than LSMC. For sufficiently small time steps, both methods yield accurate estimates when 1,000 Monte Carlo simulations are performed. However, it should be emphasized that all considered numerical approaches for pricing American-style derivatives are computationally intensive, particularly due to the combination of large path simulations and repeated regression or neural network procedures required at each exercise date. When comparing the three methods with respect to the computational time, we can conclude that the Binomial is the fastest one. The LSMC is faster than the NNMC method for the same number of time steps, but it should be mentioned that the NNMC converge faster and it needs less time steps in compare the LSMC method.

Discussing the absolute error or mean squared error with respect to the true values of the American options is beyond the scope of the paper. For that purpose, the true values from stock exchanges are necessary, and these data are not easily obtained.

## CONCLUSION

This paper presented an expanded analysis of the Neural-Network Monte Carlo method for pricing American options. The method effectively combines Monte Carlo simulation with regression-based approximation to address the early exercise feature. The NNMC method provides adequate results for the call and put options with the tested set of parameters. Numerical results demonstrate that the discretization of the holding period plays a crucial role in accuracy and convergence. On the other hand, this discretization significantly increases the computation time. While the LSMC method is more sensitive to the discretization of the holding period the options with longer maturities represent the challenging task for this method. In these cases the NNMC method comes up as one possible solution.

Future research directions include efficient parallel implementations. These developments may further enhance the applicability of NNMC methods in large-scale computational finance problems.

Also, one of the extension of this research could be the comparison of these methods on the more complex derivatives. Namely, when having only one underlying asset in the option contract, it can be expected that standard linear regression method such as LSMC performs adequately. But when the complexity of the underlying asset increases, the linearity between two time steps will be disturbed. Thus, the necessity for some more complex and reliable model arises. Following the proof of concept from this paper, the NNMC method imposes itself as an applicable method for this purpose.

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## REFERENCES

- [1] J. C. Cox, S. A. Ross, and M. Rubinstein, "Option pricing: A simplified approach," *Journal of Financial Economics*, vol. 7, no. 3, pp. 229–263, 1979.
- [2] P. Wilmott, S. Howison, and J. Dewynne, *The Mathematics of Financial Derivatives*. Cambridge University Press, 1995.
- [3] F. A. Longstaff and E. S. Schwartz, "Valuing American options by simulation: A simple least-squares approach," *Review of Financial Studies*, vol. 14, no. 1, pp. 113–147, 2001.
- [4] J. Hull, *Options, Futures, and Other Derivatives*, 10th ed. Pearson, 2018.
- [5] K. L. Judd, *Numerical Methods in Economics*. MIT Press, 1998.
- [6] J. Zhuang, D. Ding, W. Lu, X. Wu, and G. Yuan, "A Gaussian Process Based Method with Deep Kernel Learning for Pricing High-Dimensional American Options," arXiv:2311.07211, 2023.
- [7] R. Handal, K. Matoya, Y. Wang, and M. Hirano, "KANOP: A Data-Efficient Option Pricing Model using Kolmogorov-Arnold Networks". *IEEE Symposium on Computational Intelligence for Financial Engineering and Economics (CiFer)* (pp. 1-9). IEEE. March, 2025.
- [8] J. Yang, and G. Li, "Gradient-enhanced sparse hermite polynomial expansions for pricing and hedging high-dimensional american options". *SIAM Journal on Financial Mathematics*, 16(3), pp.959-987. 2025.
- [9] X. Lu, J. Luo, H. Vise, and M. White, "Distributed Least-Squares Monte Carlo for American Option Pricing," *Risks*, vol. 11, no. 8, 2023.
- [10] J. Li and A. Zhang, "American options with importance sampling and shifted regressions," *Journal of Risk and Financial Management*, vol. 14, no. 8, 2021.
- [11] M. Broadie and P. Glasserman, "Generic improvements to least-squares Monte Carlo methods with applications to optimal stopping problems," *European Journal of Operational Research*, vol. 298, pp. 1132–1144, 2022.