Simulation of Dynamic Hysteresis Loops for Toroidal Sample for Sinusoidal Shape of Magnetic Flux Density

Srđan Divac, Branko Koprivica
University of Kragujevac
Faculty of Technical Sciences Čačak
Svetog Save 65, 32000 Čačak, Serbia
srdjan.divac@ftn.kg.ac.rs, branko.koprivica@ftn.kg.ac.rs

Abstract—**The aim of this paper is to present a method for simulation of dynamic hysteresis loops of toroidal sample made of electrical steel for sinusoidal shape of magnetic flux density. Method is based on separation of dynamic magnetic field into its quasistatic, eddy current and excess magnetic field components. Quasistatic magnetic field has been calculated by means of interpolation of amplitudes and phases of quasistatic magnetic field measured at 1 Hz for sinusoidal shape of magnetic flux density. Magnetic field of eddy currents has been calculated using well-known analytical expression. Parameters for calculation of excess magnetic field have been obtained using dynamic hysteresis loops measured at frequencies of 50 Hz, 80 Hz and 100 Hz for sinusoidal shape of magnetic flux density. Method has been verified for amplitudes of magnetic flux density of 0.5 T, 0.9 T and 1.5 T. Details of the calculation procedure, measurement and verification results, as well as their adequate analysis, have been presented in this paper.**

Keywords-Simulation; Hysteresis loops; Magnetic field separation; Harmonics; Interpolation;

I. INTRODUCTION

Characterising nonlinearity of inductive element made of electric steel sheet by using magnetising curve coupled with a resistor to represent its core power losses can be very restrictive when solving magnetic circuits in time domain [1]. Hysteresis models have been researched to mitigate this problem. While very useful, more commonly used models: Preisach [2], Jilles-Atherton [3], Play model [4] and other, are often difficult to incorporate due to their complex mathematics and parameter evaluation. A simple and effective way of simulating hysteresis loops has been presented in previous work [5]. This method is based on interpolation of the amplitudes and phases of harmonic components of measured magnetic fields for known amplitudes of magnetic flux density to obtain harmonic components of magnetic field for amplitude of magnetic flux density of interest. New magnetic field waveform, calculated by summing up its newly obtained harmonic components, can be plotted against simulated ideal magnetic flux density waveform to obtain new hysteresis loop. However, presented simulation method has only been demonstrated for the frequency for which the initial loops have been measured. This paper expands the presented method to also include simulations at different frequencies.

Dynamic magnetic field can be separated into its quasistatic, eddy current and excess magnetic field components [6]. The quasistatic magnetic field component has been calculated using the simulation procedure presented in [5] and a set of measured quasistatic hysteresis loops measured at frequency of 1 Hz. Eddy current magnetic field has been derived by using its analytical expression presented in [7]. Excess magnetic field parameters have been obtained by fitting excess power loss calculated according to the procedure presented in the previous paper [8], at frequencies of 50 Hz, 80 Hz and 100 Hz and for sinusoidal shape of magnetic flux density with amplitude of 1 T. Fitting has been performed using the criteria of least root mean square deviation (RMSD [9]) between excess power losses produced by fitted and calculated excess magnetic field. These parameters have been held constant for all simulations at the respective frequency. Simulations of the hysteresis loops have been performed for sinusoidal shape of magnetic flux density with amplitudes of 0.5 T, 0.9 T and 1.5 T for all considered frequencies.

Measurements have been performed with a toroidal sample made of electrical steel sheet using a measurement method based on data acquisition and PC [10]. A set of quasistatic hysteresis loops has been measured for controlled sinusoidal shape of magnetic flux density waveform and amplitudes from 0.2 T up to 1.6 T with step of 0.2 T. Also, measurements have been made at frequencies of 1 Hz, 50 Hz, 80 Hz and 100 Hz and amplitudes of 0.5 T, 0.9 T and 1.5 T for sinusoidal shape of magnetic flux density waveform to verify the simulation method.

Details of the simulation procedure, comparison of measured and simulated hysteresis loops, as well as the adequate discussion, have also been presented in this paper.

II. SIMULATION PROCEDURE

Dynamic magnetic field waveform $H(t)$ can be represented as a sum of its three components - quasistatic $H_{qs}(t)$, eddy current $H_{eddy}(t)$ and excess $H_{exc}(t)$ [6]:

$$H(t) = H_{qs}(t) + H_{eddy}(t) + H_{exc}(t) . \qquad (1)$$

This study was supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia, and these results are parts of the Grant No. 451-03-68/2022-14/200132 with University of Kragujevac - Faculty of Technical Sciences Čačak.

Waveform of $H_{qs}(t)$ for the sinusoidal shape of magnetic flux density waveform $B(t)$ can be calculated by summing up its harmonic components for which amplitudes and phases have been obtained for amplitude of $B(t)$ of interest. These amplitudes and phases can be calculated by interpolating the amplitudes and phases of measured quasistatic magnetic fields for known amplitudes of measured $B(t)$ to the amplitude of $B(t)$ of interest, as presented in [5]. Quasistatic measurements should be performed at very low frequency so that the measured quasistatic loops and static loops for the used sample made of electrical steel are in good agreement as much as possible [11].

Magnetic field of eddy currents can be calculated as follows [7]:

$$H_{eddy}(t) = \frac{\sigma d^2}{12} \frac{\mathrm{d}B(t)}{\mathrm{d}t}, \qquad (2)$$

where σ is the conductivity of steel sheet, d is its thickness and t is the time.

Excess magnetic field can be calculated as [7]:

$$H_{exc}(t) = \frac{n_0 V_0}{2} \left(\sqrt{1 + \frac{4\sigma G S}{n_0^2 V_0} \frac{\mathrm{d}B(t)}{\mathrm{d}t}} - 1 \right), \qquad (3)$$

where G is equal to 0.1356, S is the area of the sample cross-section and n_0 and V_0 are phenomenological parameters of the material.

Dynamic magnetic field waveform $H(t)$ for the sinusoidal shape and amplitude of magnetic flux density waveform of interest can be obtained by calculating (2) and (3) and adding them up to calculated $H_{qs}(t)$, according to (1). Lastly, new hysteresis loop can be formed by using calculated results for $H(t)$ and the simulated sinusoidal waveform of $B(t)$.

III. MEASUREMENT SETUP

Measurements of magnetic field $H(t)$ and magnetic flux density $B(t)$ waveforms have been performed by using measurement method based on data acquisition and PC [10]. Measurement setup is shown in Fig. 1. Measurements have been made for toroidal sample made of electrical steel sheet 27PH100 (manufactured by POSCO) for controlled sinusoidal shape of $B(t)$. Parameters of the used sample (number of turns of magnetising N_1 and induction N_2 coils, cross-section area S and magnetic path length l) can be found in [5].

Measurement setup consists of voltage-controlled AC power source, shunt resistor R=0.5 Ω, toroidal sample, acquisition card and PC. Voltage controlled AC power source has been formed by connecting audio amplifier CROWN Xli 2500, isolation transformer 230/30 V/V, and acquisition card NI PCI-6259. Output of this system is controlled by PC with LabVIEW application. Voltage over the shunt resistor and induced voltage in induction coil of toroidal sample have been measured by the same acquisition card using LabVIEW application. Also, this application is used to control the shape and amplitude of $B(t)$, calculated using (5), according to the set magnetic flux density waveform.

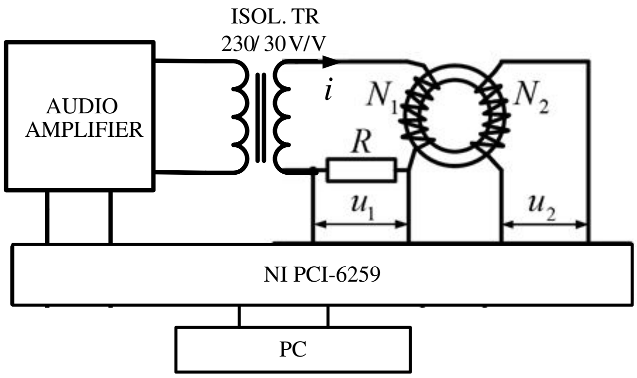

Figure 1. Measurement setup.

Measurements of $u_1(t)$ and $u_2(t)$ have been performed with 1000 data points per waveform period.

Magnetising current $i(t)$ has been measured indirectly and calculated as a ratio of voltage measured over the shunt resistor $u_1(t)$ and its resistance R. The result has been used for calculation of $H(t)$ using Ampere's law [12], as follows:

$$H(t) = \frac{N_1 i(t)}{l} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right]. \qquad (4)$$

In accordance with Faraday's law of electromagnetic induction, the voltage measured over the induction coil $u_2(t)$ is proportional to the rate of change of $B(t)$, so $B(t)$ can be calculated as:

$$B(t) = -\frac{1}{N_2 S} \int_0^t u_2(t)\,\mathrm{d}t \quad [\mathrm{T}]. \qquad (5)$$

MEASUREMENT RESULTS

A set of quasistatic hysteresis loops has been measured at frequency of 1 Hz for controlled sinusoidal shape of $B(t)$ at amplitudes ranging from 0.2 T up to 1.6 T with step of 0.2 T. These loops are presented in Fig. 2.

Four sets of hysteresis loops have also been measured to verify the proposed simulation method. These loops have been measured at frequencies of 1 Hz, 50 Hz, 80 Hz and 100 Hz for amplitudes of sinusoidal $B(t)$ of 0.5 T, 0.9 T and 1.5 T.

Additionally, $H(t)$ used for obtaining of excess field parameters have been measured for sinusoidal $B(t)$ with amplitude of 1 T at frequencies of 50 Hz, 80 Hz and 100 Hz.

IV. SIMULATION RESULTS AND ANALYSIS

Dynamic hysteresis loops have been simulated for amplitudes of $B(t)$ of 0.5 T, 0.9 T and 1.5 T and frequencies of 50 Hz, 80 Hz and 100 Hz. Quasistatic magnetic field waveforms for sinusoidal shape of $B(t)$ have been calculated using measured quasistatic loops at 1 Hz and simulation procedure presented in [5]. Total number of N=35 harmonics and 1000 data points have been used in simulations. Contribution of the harmonics of order higher than N has been found negligible for amplitude of 1.5 T. It is possible to use 2 to 3 times less harmonics for lower amplitudes.

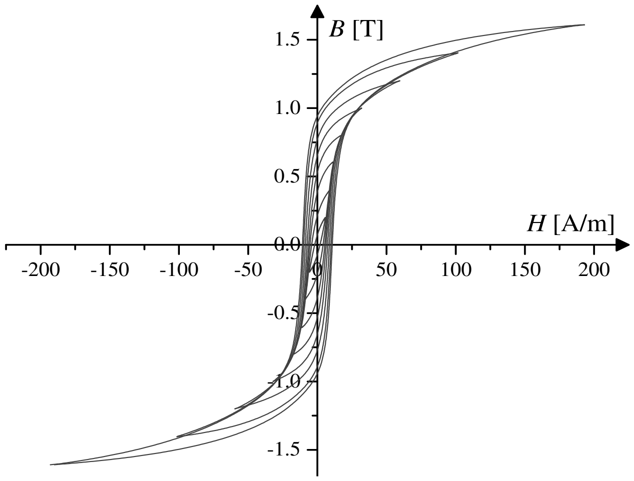

Figure 2. Measured quasistatic hysteresis loops at 1 Hz for sinusoidal $B(t)$.

Magnetic flux density waveforms at 1 Hz for all its considered amplitudes have been simulated using ideal sinusoidal functions with 1000 data points. These waveforms have been plotted versus calculated magnetic field waveforms to form simulated quasistatic hysteresis loops. A comparison of simulated and measured quasistatic hysteresis loops for all considered amplitudes of $B(t)$ is shown in Fig. 3. A good agreement has been found for the compared hysteresis loops. Reduced number of measurement data points has been shown in Fig. 3, as well as in all the following figures, using a built-in function (*Skip points*) of Origin program. The reduction is purely visual and has no effect on the results presented.

Toroidal sample has been made of electrical steel sheet with conductivity of σ=2083 kS/m and thickness of d=0.27 mm. Eddy currents magnetic field $H_{eddy}(t)$ has been calculated using (2) and these parameters.

Parameters n_0 and V_0 in (3) have been obtained by fitting the excess power loss for sinusoidal shape of $B(t)$ at 1 T. Fitting has been performed for all considered frequencies using the criteria of least RMSD between excess power losses produced by fitted and calculated excess magnetic field [8]. Both parameters have been kept constant in all simulations performed at the frequency for which they have been found.

Parameter V_0=0.08 A/m has been found to be constant for all frequencies, while n_0 has been varied according to the frequency of magnetic flux density waveform. It has been found to be n_0=908, n_0=1098 and n_0=1206 for frequencies of 50 Hz, 80 Hz and 100 Hz, respectively. Comparisons of calculated and measured dynamic hysteresis loops for each of the considered frequencies are shown in Figs. 4-6.

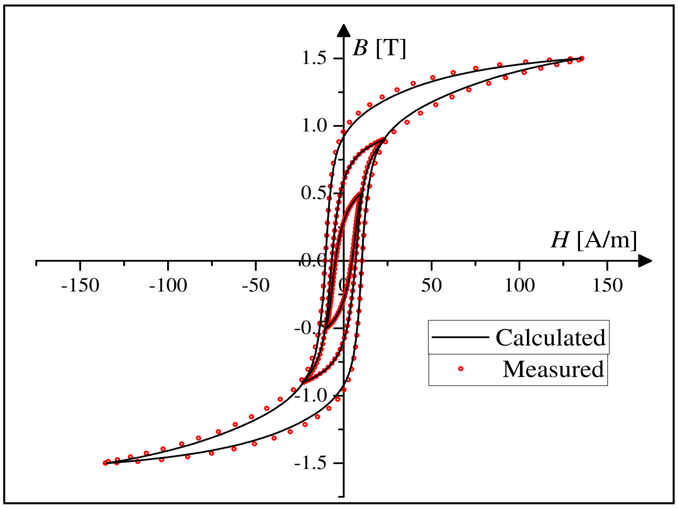

Figure 3. Comparison of simulated and measured hysteresis loops at 1 Hz.

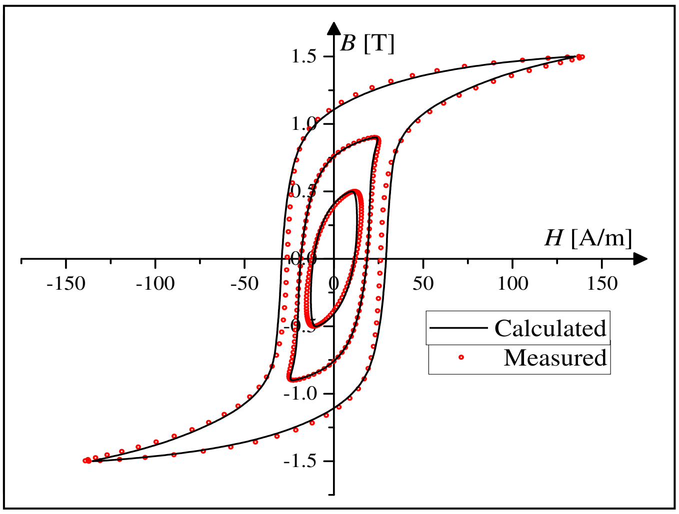

Figure 4. Comparison of simulated and measured hysteresis loops at 50 Hz.

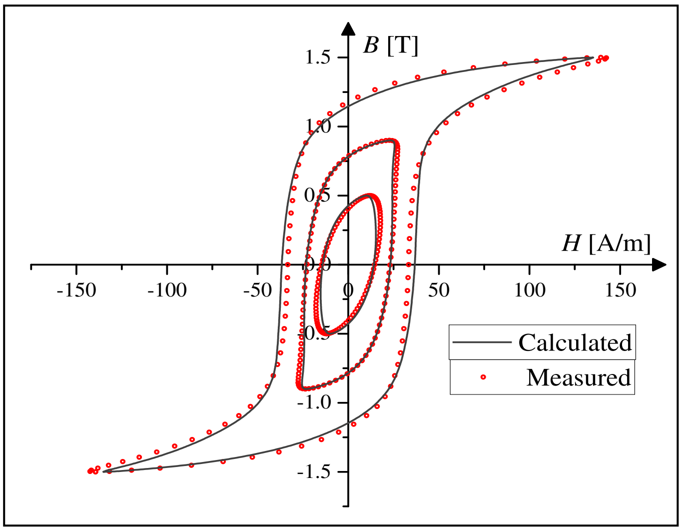

Figure 5. Comparison of simulated and measured hysteresis loops at 80 Hz.

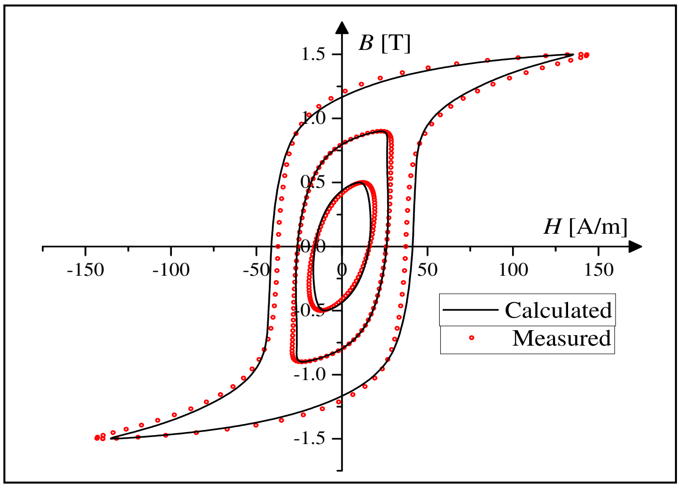

Figure 6. Comparison of simulated and measured hysteresis loops at 100 Hz.

Overall, a good agreement of measured and simulated results has been found. The best agreement has been found for the amplitude of $B(t)$ of 0.9 T, followed by 1.5 T and lastly 0.5 T. This is mostly due to the fact that excess magnetic field parameters have been obtained for 1 T which is closest to 0.9 T. The calculation procedure could be optimised further by varying n_0 both with frequency and amplitude of $B(t)$. However, such optimisation is beyond the scope of this paper and it will be addressed in the future publications.

Relative deviation between areas of simulated and measured hysteresis loops has been calculated as:

$$\delta[\%] = \frac{S_{sim} - S_m}{S_m} 100 , \tag{6}$$

where S_{sim} and S_m are the areas of simulated and measured loops, respectively.

Relative deviations of hysteresis loops areas for considered amplitudes and frequencies of $B(t)$ are given in Table I. According to good agreements between measured and simulated hysteresis loops for sinusoidal $B(t)$, presented simulation method could suitable for solving magnetic problems in steady state time domain. Use of presented simulation process has not been tested for simulation of transient processes.

TABLE I. RELATIVE DEVIATIONS OF HYSTERESIS LOOPS AREAS

	δ [%]		
f [Hz] / B_{max} [T]	50	80	100
0.5	-7.29	-5.91	-4.9
0.9	-2.52	-2.64	-2.96
1.5	-0.14	-1.6	-0.90

V. CONCLUSION

A method for simulation of dynamic hysteresis loops for frequency of interest and sinusoidal shape of magnetic flux density has been presented in this paper. Method is based on separation of magnetic field into its quasistatic, eddy current and excess component. Quasistatic magnetic field has been calculated by interpolation of the amplitudes and phases of the quasistatic magnetic field measured at 1 Hz, for sinusoidal shape of magnetic flux density of amplitude of interest. Measurements of quasistatic loops have been performed at 1 Hz for amplitudes ranging from 0.2 T up to 1.6 T with step of 0.2 T. Eddy current magnetic field has been calculated analytically, whereas excess magnetic field has been calculated using parameters obtained for fields measured at 1 T and frequencies of 50 Hz, 80 Hz and 100 Hz. Simulations were made for each frequency of interest - at 0.5 T, 0.9 T and 1.5 T.

Verification of the simulation results has been performed by comparing the simulated and measured hysteresis loops for each of the considered frequencies and amplitudes of magnetic flux density. Overall, a good agreement has been found between all the results compared.

This simulation method could be suitable for solving steady-state magnetic problems in time domain. Further development of the method will concern its optimisation of calculation of parameters n_0 and V_0, as well as its use for simulation of hysteresis loops for nonsinusoidal shape of magnetic flux density.

REFERENCES

[1] A. Rezaei-Zare and R. Iravani, “On the Transformer Core Dynamic Behavior During Electromagnetic Transients”, IEEE Transactions on Power Delivery, Vol. 25, No. 3, pp. 1606-1619, July 2010.

[2] I. D. Mayergoyz, “Mathematical Models of Hysteresis and Their Applications”, Elsevier Science, New York, USA, 2003.

[3] A. R. P. J. Vijn, O.Baas and E. Leepelaars, “Parameter Estimation for the Jiles–Atherton Model in Weak Fields”, IEEE Transactions on Magnetics, Vol. 56, No. 4, April 2020.

[4] Y. Hane and K. Nakamura, “Dynamic Hysteresis Modeling for Magnetic Circuit Analysis by Incorporating Play Model and Causer’s Equivalent Circuit Theory”, IEEE Transactions on Magnetics, Vol. 56, No. 8, Aug. 2020.

[5] S. Divac and B. Koprivica, “Simulation of Hysteresis Loops by Interpolation of Harmonic Components of Magnetic Field”, 65th ETRAN Conf., 8-10. Sept. 2021, Ethno Village Stanišići, Bijeljina, Republic of Srpska, Bosnia and Hercegovina, pp. 174-179, (in Serbian).

[6] B. Koprivica, M. Rosić, K and Chwastek, “Time Domain Analysis of Excess Loss in Electrical Steel”, Serbian Journal of Electrical Engineering, Vol. 16, No. 3, pp. 419-437, Oct. 2019.

[7] G. Bertotti, “Hysteresis in Magnetism. for Physicists, Material Scientists and Engineers”, Academic Press, New York, USA, 1998.

[8] B. Koprivica and S. Divac, “Analysis and Modeling of Instantaneous Magnetizing Power of Ferromagnetic Core in Time Domain”, IEEE Magnetic Letters, Vol. 12, p. 2103505, Sept. 2021.

[9] C. D. Schunn and D. Wallach, “Evaluating Goodness-of-Fit in Comparison of Models to Data, pp. 115-135, 2005. In W. Tack (Ed.), Psychologie der Kognition: Reden and Vorträge anlässlich der Emeritierung von Werner Tack, Saarbrueken, Germany, University of Saarland Press.

[10] B. Koprivica, A. Milovanović and M. Đekić, “Effects of Wound Toroidal Core Dimensional and Geometrical Parameters on Measured Magnetic Properties of Electrical Steel”, Serbian Journal of Electrical Engineering, Vol. 10, No. 3, pp. 459-471, Oct. 2013.

[11] M. De Wulf, L. Dupre and J. Melkebeek, “Quasistatic Measurements for Hysteresis Modeling”, Journal of Applied Physics, Vol. 87, No. 9, pp. 5239-5241, May 2000,

[12] K. L. Kaiser, “Electromagnetic Compatibility Handbook”, CRC Press, New York, USA, 2004.