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Performance of wireless communication system with diversity receiver operating over mixed Rician and Nakagami-*m* multipath fading channel

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Abstract—**In this paper wireless communication radio system with maximal ratio combining (MRC) and selection combining (SC) receiver in the presence Rician and Nakagami-*m* fading are considered. Received signal in the first branch of receiver experiences Rician fading and the received signal in the second branch experiences Nakagami-*m* fading. Closed form expressions for probability density function (PDF), cumulative distribution function (CDF) and level crossing rate (LCR) of radio receiver output random processes are efficiently evaluated. The influence of Rician κ factor and Nakagami-*m* short term fading severity parameter on level crossing rate is analyzed and discussed. Derived expressions for level crossing rate can be used for evaluation average fade duration of proposed wireless communication system.**

Keywords-MRC; SC; Rican fading; Nakagami-m fading, PDF; CDF; LCR

I. INTRODUCTION

Rician short term fading and Nakagami-*m* short term fading degrade outage probability, average bit error probability and average fade duration of wireless communication system and limit channel capacity [1-3]. In this paper is considered scenario when signal at the first antenna propagates in line-of-sight multipath fading channel and signal at the second antenna propagates over non line-of-sight short term fading channel resulting in the signal at the first antenna experiences Rican fading and the signal at the second antenna experiences Nakagami-*m* fading. Short term fading effects can be reduced by using diversity techniques. In this paper MRC diversity technique and SC diversity technique are used to mitigate Rician fading effects and Nakagami-*m* fading effects on system performance. Square of signal envelope at output of MRC is equal to sum of squares signal envelope at inputs of MRC receiver. SC receiver selects branch with the highest signal to provide service to user. Signal envelope at the first branch is described by using Rician distribution and signal envelope at the second antenna is described by using Nakagami-*m* distribution. Probability density function receiver output signal is derived by using definition of output signal for MRC and SC receiver. Cumulative distribution function is calculated from probability density function. Level crossing rate is calculated as average values of the first derivative of receiver output signal random process. Complex channel where one signal envelope is described by using Rician distribution and on other signal envelope is described with Nakagami-*m* distribution can be named with Rician-Nakagami-*m* channel and distribution can be named with Rician-Nakagami-m distribution. When Rician factor goes to zero and Nakagami-*m* shaping factor goes to one Rician-Nakagami-*m* channel becomes Rayleigh-Rayleigh channel. Rician-Nakagami-*m* distribution reduces to Rayleigh-Nakagami-*m* distribution for κ=0 and Rician-Rayleigh distribution is derived from Rican-Nakagami-*m* distribution for *m*=1 [4-7].

There are more works in open technical literature considering performance the first order and second order of wireless communication system in the presence Rician short term fading and Nakagami-*m* short term fading. In paper [8], wireless communication system with dual SC receiver operating over Nakagami-*m* multipath fading channel in the presence Nakagami-*m* co-channel interference is studied. PDF, CDF and moments of output signal are evaluated and by using these formulas outage probability and average bit error probability for several modulation schemes are efficiently calculated. Average level crossing rate and average fade duration of macrodiversity reception with macrodiversity SC receiver and two microdiversity MRC receivers operating over Gamma shadowed, Rician multipath fading channel are evaluated in work [9]. The second order statistical measures macrodiversity system with macrodiversity SC receiver and two microdiversity MRC receivers in the presence Gamma long term fading and Nakagami-*m* short term fading are studied in paper [10]. In paper [11], radio SC receiver operating over correlated Rician multipath fading channel in the presence of correlated co-channel interference subjected to Rician short term fading is considered.

In this paper SC receiver and MRC receiver in the presence Rician-Nakagami-*m* multipath fading are studied. Closed form expressions for probability density function, cumulative distribution function and average level crossing rate are calculated. By using these expressions, outage probability, bit error probability, channel capacity and average fade duration of proposed receivers can be evaluated. To the best author's knowledge, performance of wireless communication system in the presence Rician-Nakagami-*m* fading is not analyzed in open technical literature.

II. STATISTICS OF MRC RECEIVER OPERATING OVER RICIAN-NAKAGAMI-M MULTIPATH FADING CHANNEL

Model MRC receiver considered in this paper is shown in Figure 1.

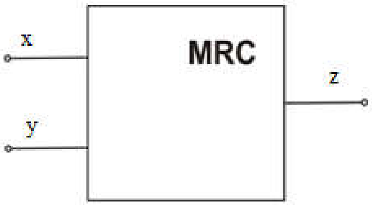

Figure 1. Maximal ratio combining receiver

Signal envelopes at inputs of MRC receiver are denoted with *x* and *y* and signal envelope at output of MRC receiver is denoted with *z*. MRC receiver provides the best performance but is complex for practical implementation. Squared *z* random variable can be calculated as sum of squared *x* random variable and squared *y* random variable. Random variable *x* follows Rician distribution:

$$p_x(x)=\frac{2(\kappa+1)}{e^{\kappa}\Omega_1}e^{\frac{-(\kappa+1)}{\Omega_1}x^2}I_0\left(2\sqrt{\frac{\kappa(\kappa+1)}{\Omega_1}}\right)=$$
$$=\frac{2(\kappa+1)}{e^{\kappa}\Omega_1}\sum_{i_1=0}^{\infty}\left(\frac{\kappa(\kappa+1)}{\Omega_1}\right)^{i_1}\frac{1}{(i_1!)^2}x^{2i_1+1}e^{\frac{-(\kappa+1)}{\Omega_1}x^2},\quad x\geq 0, \tag{1}$$

where κ is Rician factor and Ω_1 is signal envelope average power. Rician factor can be calculated as ratio of dominant component power and scattering component power. Signal envelope *y* has Nakagami-*m* distribution:

$$p_y(y)=\frac{2}{\Gamma(m)}\left(\frac{m}{\Omega_2}\right)^m y^{2m-1}e^{-\frac{m}{\Omega_2}y^2},\quad y\geq 0, \tag{2}$$

where *m* is Nakagami-m shape parameter and Ω_2 is signal envelope average power. Squared random variable *z* is:

$$z^2=x^2+y^2=x_1^2+x_2^2+\sum_{j=1}^{m}\left(y_{1j}^2+y_{2j}^2\right), \tag{3}$$

where x_1 and x_2 Gaussian random variable with mean A_i and variances σ^2. Random variable y_{1j} and y_{2j}, j=1, 2, …, *m* are independent zero mean Gaussian random variable with variances σ^2. Random variable z has *κ-μ* distribution. Rician factor κ of Rician random variable *x* is:

$$\kappa=\frac{A^2}{2\sigma^2},\quad \Omega_2=2m\sigma^2\ \text{and}$$
$$\Omega_1=A^2+2\sigma^2=\kappa 2\sigma^2+2\sigma^2=2\sigma^2(\kappa+1). \tag{4}$$

Rician factor κ_1 of *κ-μ* random variable *z* is:

$$\kappa_1=\frac{A^2}{2\sigma^2+2m\sigma^2}=\frac{A^2}{2\sigma^2(1+m)}=\frac{\kappa}{(1+m)}. \tag{5}$$

Random variable *z* has *κ-μ* distribution [12]:

$$p_z(z)=\frac{2\mu(\kappa_1+1)^{\frac{\mu+1}{2}}z^{\mu}}{\kappa_1^{\frac{\mu-1}{2}}e^{\kappa_1\mu}\Omega_3^{\frac{\mu+1}{2}}}e^{\frac{-\mu(\kappa_1+1)}{\Omega_3}z^2}I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(\kappa+1)}{\Omega_3}}\right)=$$
$$=\frac{2\mu(\kappa_1+1)^{\frac{\mu+1}{2}}}{\kappa_1^{\frac{\mu-1}{2}}e^{\kappa_1\mu}\Omega_3^{\frac{\mu+1}{2}}}\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{\kappa_1(\kappa_1+1)}{\Omega_3}}\right)^{2i_2+\mu-1} \tag{6}$$
$$\times\frac{1}{i_2!\Gamma(i_2!+\mu)}z^{2i_2+2\mu-1}e^{\frac{-\mu(\kappa_1+1)}{\Omega_3}z^2}.$$

Parameter *m* is:

$$\mu=m+1, \tag{7}$$

and average value of *z* is:

$$\Omega_3=\Omega_1+\Omega_2. \tag{8}$$

Cumulative distribution function of *z* is [12]:

$$F_z(z)=\int_0^z dt\,p_z(t)=\frac{2\mu(\kappa_1+1)^{\frac{\mu+1}{2}}}{\kappa_1^{\frac{\mu-1}{2}}e^{\kappa_1\mu}\Omega_3^{\frac{\mu+1}{2}}}\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{\kappa_1(\kappa_1+1)}{\Omega_3}}\right)^{2i_2+\mu-1}$$
$$\times\frac{1}{i_2!\Gamma(i_2!+\mu)}\frac{1}{2}\left(\frac{\Omega_3}{\kappa_1(\kappa_1+1)}\right)^{i_2+\mu}\gamma\left(i_2+\mu,\frac{\mu(\kappa_1+1)}{\Omega_3}z^2\right). \tag{9}$$

Join probability density function of z and $\dot{z}$ is:

$$p_{z\dot{z}}\left(z\dot{z}\right)=p_z\left(z\right)p_{\dot{z}}\left(\dot{z}\right)=p_z\left(z\right)\frac{1}{\sqrt{2\pi}\sigma_{\dot{z}}}e^{-\frac{\dot{z}^2}{2\sigma_{\dot{z}}^2}}, \qquad (10)$$

where:

$$\sigma_{\dot{z}}^2=\pi f_m^2\frac{\Omega_3}{\kappa_1\left(\kappa_1+1\right)}. \qquad (11)$$

Average level crossing rate of z is:

$$\begin{aligned}N_z&=\int_0^\infty dz\,\dot{z}\,p_{z\dot{z}}\left(z\dot{z}\right)=p_z\left(z\right)\int_0^\infty dz\,\dot{z}\,p_{\dot{z}}\left(\dot{z}\right)=p_z\left(z\right)\frac{\sigma_{\dot{z}}}{\sqrt{2\pi}}=\\&=\frac{2\mu^{1/2}\left(\kappa_1+1\right)^{\frac{\mu}{2}}\pi f_m}{\kappa_1^{\frac{\mu-1}{2}}e^{\kappa_1\mu}\Omega_3^{\mu/2}\sqrt{2\pi}}\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{\kappa_1\left(\kappa_1+1\right)}{\Omega_3}}\right)^{2i_2+\mu-1}\\&\times\frac{1}{i_2!\Gamma\left(i_2!+\mu\right)}z^{2i_2+2\mu-1}e^{\frac{-\mu\left(\kappa_1+1\right)}{\Omega_3}z^2}.\end{aligned} \qquad (12)$$

III. PERFORMANCE SC RECEIVER OPERATING OVER RICIAN-NAKAGAMI-*M* MULTIPATH FADING CHANNEL

Model SC receiver considered in this paper is presented in Figure 2.

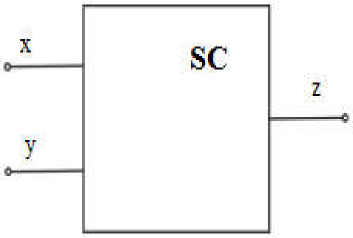

Figure 2. Selection combining receiver

Random variable x follows Rician distribution:

$$p_x\left(x\right)=\frac{2\left(\kappa-1\right)}{e^{\kappa}\Omega_1}\sum_{i_1=0}^{\infty}\left(\frac{\kappa\left(\kappa-1\right)}{\Omega_1}\right)^{i_1}\frac{1}{\left(i_1!\right)^2}x^{2i_1+1}e^{\frac{-\left(\kappa-1\right)}{\Omega_1}x^2}, x\ge 0. \qquad (13)$$

Cumulative distribution function of x is [12]:

$$\begin{aligned}F_x\left(x\right)&=\int_0^x dt\,p_x\left(t\right)=\frac{2\left(\kappa+1\right)}{e^{\kappa}\Omega_1}\sum_{i_1=0}^{\infty}\left(\frac{\kappa\left(\kappa+1\right)}{\Omega_1}\right)^{i_1}\frac{1}{\left(i_1!\right)^2}\\&\times\frac{1}{2}\left(\frac{\Omega_1}{\kappa+1}\right)^{i_1+1}\gamma\left(i_1+1,\frac{\kappa+1}{\Omega_1}x^2\right).\end{aligned} \qquad (14)$$

Random variable y follows Nakagami-m distribution:

$$p_y\left(y\right)=\frac{2}{\Gamma\left(m\right)}\left(\frac{m}{\Omega_2}\right)^m y^{2m-1}e^{-\frac{m}{\Omega_2}y^2}, \quad y\ge 0. \qquad (15)$$

Cumulative distribution function of y is:

$$F_y\left(y\right)=\frac{1}{\Gamma\left(m\right)}\gamma\left(m,\frac{m}{y}y^2\right), \quad y\ge 0. \qquad (16)$$

Probability density function of SC receiver output signal is:

$$\begin{aligned}p_z\left(z\right)&=p_x\left(z\right)F_y\left(z\right)+p_y\left(z\right)F_x\left(z\right)=\\&=\frac{2\left(\kappa+1\right)}{e^{\kappa}\Omega_1}\sum_{i_1=0}^{\infty}\left(\frac{\kappa\left(\kappa+1\right)}{\Omega_1}\right)^{i_1}\frac{1}{\left(i_1!\right)^2}z^{2i_1+1}e^{\frac{-\kappa+1}{\Omega_1}z^2}\\&\times\frac{1}{\Gamma\left(m\right)}\gamma\left(m,\frac{m}{y}z^2\right)+\frac{2}{\Gamma\left(m\right)}\left(\frac{m}{\Omega_2}\right)^m z^{2m-1}\\&\times e^{-\frac{m}{\Omega_2}z^2}\frac{2\left(\kappa+1\right)}{e^{\kappa}\Omega_1}\sum_{i_1=0}^{\infty}\left(\frac{\kappa\left(\kappa+1\right)}{\Omega_1}\right)^{i_1}\frac{1}{\left(i_1!\right)^2}\\&\times\frac{1}{2}\left(\frac{\Omega_1}{\kappa+1}\right)^{i_1+1}\gamma\left(i_1+1,\frac{\kappa+1}{\Omega_1}z^2\right).\end{aligned} \qquad (17)$$

Cumulative distribution function of SC receiver output signal is:

$$\begin{aligned}F_z\left(z\right)&=F_x\left(z\right)F_y\left(z\right)=\frac{2\left(\kappa+1\right)}{e^{\kappa}\Omega_1}\sum_{i_1=0}^{\infty}\left(\frac{\kappa\left(\kappa+1\right)}{\Omega_1}\right)^{i_1}\frac{1}{\left(i_1!\right)^2}\\&\times\frac{1}{2}\left(\frac{\Omega_1}{\kappa+1}\right)^{i_1+1}\gamma\left(i_1+1,\frac{\kappa+1}{\Omega_1}z^2\right)\frac{1}{\Gamma\left(m\right)}\gamma\left(m,\frac{m}{y}z^2\right).\end{aligned} \qquad (18)$$

Level crossing rate of SC receiver output signal process is:

$$\begin{aligned}N_z&=\int_0^\infty dz\,\dot{z}\,p_{z\dot{z}}\left(z\dot{z}\right)=\\&=F_y\left(z\right)\int_0^\infty dz\,\dot{z}\,p_{x\dot{x}}\left(x\dot{x}\right)+F_x\left(z\right)\int_0^\infty dz\,\dot{z}\,p_{y\dot{y}}\left(y\dot{y}\right)=\\&=F_y\left(z\right)p_x\left(z\right)\frac{\sigma_{\dot{x}}}{\sqrt{2\pi}}+F_x\left(z\right)p_y\left(z\right)\frac{\sigma_{\dot{y}}}{\sqrt{2\pi}},\end{aligned} \qquad (19)$$

where:

$$\sigma_{\dot{x}}^2=\pi f_m^2\frac{\Omega_1}{\kappa+1}, \quad \sigma_{\dot{y}}^2=\pi f_m^2\frac{\Omega_2}{m}, \qquad (20)$$

After substituting, the expression for N_z becomes:

$$
\begin{aligned}
N_z =& \frac{\pi f_m}{\sqrt{2\pi}} \left(\frac{1}{\Gamma(m)} \gamma\left(m, \frac{m}{y} z^2 \right) \frac{2(\kappa+1)^{1/2}}{e^{\kappa}\Omega_1^{1/2}} \sum_{i_1=0}^{\infty} \left(\frac{\kappa(\kappa+1)}{\Omega_1} \right)^{i_1} \right. \\
& \times \frac{1}{(i_1!)^2} z^{2i_1+1} e^{\frac{-\kappa+1}{\Omega_1} z^2} + \frac{2(\kappa+1)}{e^{\kappa}\Omega_1} \sum_{i_1=0}^{\infty} \left(\frac{\kappa(\kappa+1)}{\Omega_1} \right)^{i_1} \\
& \times \frac{1}{(i_1!)^2} \frac{1}{2} \left(\frac{\Omega_1}{\kappa+1} \right)^{i_1+1} \gamma\left(i_1+1, \frac{\kappa+1}{\Omega_1} z^2 \right) \frac{2}{\Gamma(m)} \\
& \left. \times \left(\frac{m}{\Omega_2} \right)^{m-1/2} z^{2m-1} e^{-\frac{m}{\Omega_2} z^2} \right).
\end{aligned}
\tag{21}
$$

IV. NUMERICAL RESULTS

In this section, the behavior of cumulative distribution function and level crossing rate measures at the output of dual MRC and SC combiners illustrated for different values of system's parameters. The numerical results are presented graphically to show the influence of fading parameters on cumulative distribution function and level crossing rate of wireless communication systems with MRC and SC diversity receiver.

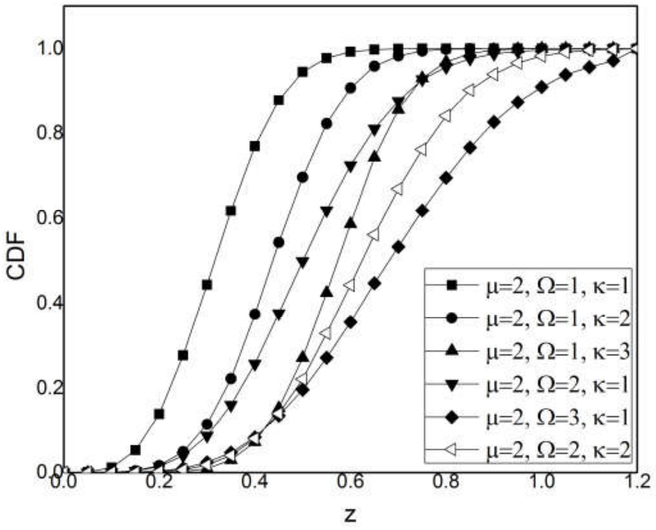

Figure 3. CDF of MRC receiver for different parameters Ω and κ

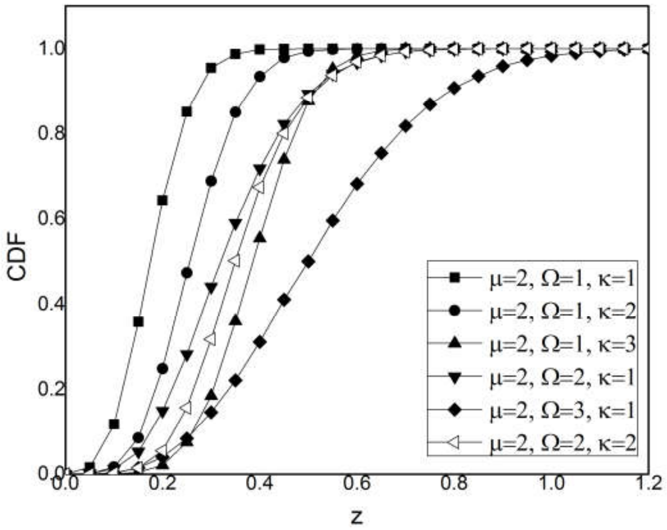

Figure 4. CDF of of SC receiver for different parameters Ω and κ

In Fig. 3 and Fig. 4 shows the cumulative distribution function and influence parameter average power Nakagami-*m* fading and Rician factor using different diversity technique. In Fig 5 and Fig. 6 shows the level crossing rate in the presence mixed Rician-Nakagami-*m* fading channel for MRC and SC combiners.

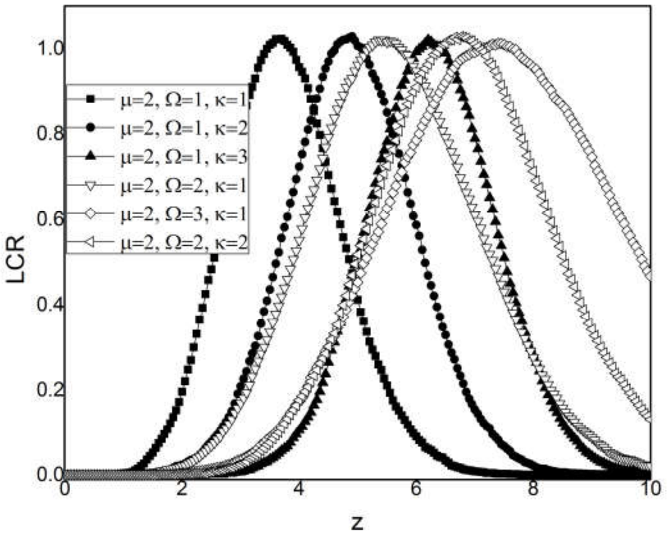

Figure 5. Level crossing rate of MRC receiver for different parameters Ω and κ

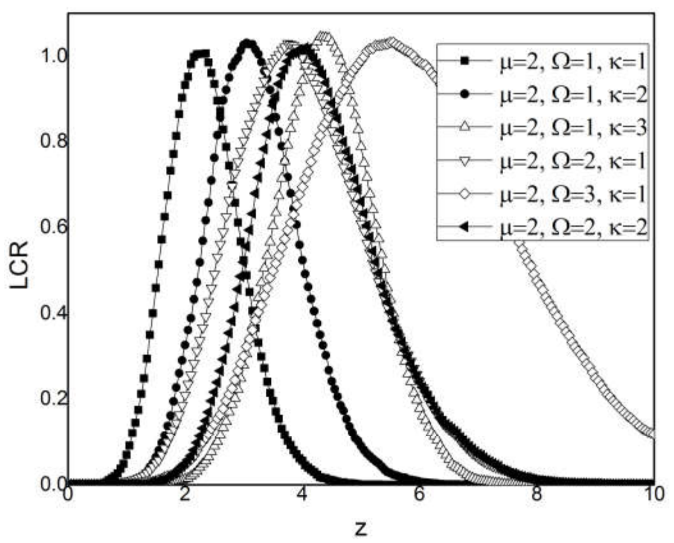

Figure 6. Level crossing rate of SC receiver for different parameters Ω and κ

V. CONCLUSION

In this paper wireless communication system with SC and MRC receiver with two inputs is considered. Signal at one input propagates in line-of-sight multipath fading channel and signal at another input propagates in non line-of-sight short term fading channel. Signal envelope at the first input experiences, Rician multipath fading and signal envelope at the second input experiences Nakagami-*m* multipath fading. This complex channel is denoted with Rician-Nakagami-*m* channel. Rayleigh-Rayleigh, Rayleigh-Nakagami-m and Rician-Rayleigh complex channels are special cases of Rician-Nakagami-*m* channel. Two cases are analysed in this paper. In the first cases MRC receiver is used to reduce Rician-Nakagami-*m* fading effects on system performance and in the second cases SC receiver is used to mitigate Rician-Nakagami-*m* multipath fading effects on outage probability and bit error probability. Closed form expressions for probability density function, cumulative distribution function and average level crossing rate of wireless system receiver output signal are calculated PDF, CDF and LCR of wireless communication system operating over Rayleigh-Rayleigh, Rayleigh-Nakagami-*m* or Rician-Rayleigh complex fading channels can be calculated from derived expressions in this paper. The influence of Rician factor and Nakagami-*m* multipath fading severity parameter on average level crossing rate is analyzed and discussed. Average level crossing rate takes lower values for higher values of Rician factor and Nakagami-*m* shaping parameter.

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