

# Level crossing rate of macrodiversity reception operating over mixed shadowed small scale fading channel

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**Abstract**—Macrodiversity (MACD) reception with macrodiversity selection combining (SC) receiver and two microdiversity (MICD) maximal ratio combining (MRC) receivers in the presence short term fading and long term fading is considered. The received signal at input of the first microdiversity experiences Nakagami- $m$  fading and Gamma long term fading while received signal at the input of the second is subjected to  $\kappa$ - $\mu$  short term fading and Gamma long term fading. Level crossing rates at outputs of microdiversity MRC receivers are evaluated and these expressions are used for evaluation of level crossing rate of macrodiversity SC receiver output random process. The influence of Nakagami- $m$  short term fading severity parameter, the  $\kappa$ - $\mu$  short term fading severity parameter and Gamma long term fading correlation coefficient on level crossing rate are analyzed.

**Keywords**-MACD; MICD; Nakagami- $m$ ;  $\kappa$ - $\mu$ , Gamma; LCR

## I. INTRODUCTION

Macrodiversity (MACD) technique can be used in cellular wireless communication system to reduce short term fading effects and long term fading effects on system performance [1-3].

In this paper MACD system has MACD SC receiver and two microdiversity (MICD) MRC receivers. MACD SC receiver selects MICD receiver with higher signal envelope average power at inputs to enable service to user resulting in long term fading effects reduction. MICD MRC receivers adds signal envelopes resulting in short term fading reduction. MICD MRC receivers combine signals with multiple antennas at base station while MACD SC receiver combines signals with two or more base stations. In this paper, the case when Nakagami- $m$  fading is presented at the first MICD MRC receiver and  $\kappa$ - $\mu$  multipath fading is presented at the second

MICD MRC receiver is considered. Nakagami- $m$  short term fading is general fading. For  $m=1$ , Nakagami- $m$  distribution reduces to Rayleigh distribution, and for  $m=0.5$ , one sided Gaussian distribution can be derived from Nakagami- $m$  distribution. When parameter  $m$  goes to infinity Nakagami- $m$  short term fading channel becomes no fading channel. The  $\kappa$ - $\mu$  distribution has two parameters. The parameter  $\kappa$  is Rician factor which can be calculated as ratio of dominant component power and scattering components power and the parameter  $\mu$  is in relation to the number of clusters in propagation environment [4-6].

The second order performance measures of wireless mobile communication radio systems are level crossing rate and average fade duration. Level crossing rate is defined as the number of crossings at determined level of random process and can be calculated as average value of the first derivative of random process. Average fade duration of wireless communication system is defined as average time that random process falls below of the specified value and can be calculated as ratio of outage probability and level crossing rate. Outage probability is defined as the probability that signal is lower than threshold [7-9].

There are more works in open technical literature considering level crossing rate and average fade duration in wireless communication systems. In paper [10], MACD system with MACD SC receiver and two MICD MRC receivers operating over Gamma shadowed Nakagami- $m$  short term fading channels is considered and average level crossing rate and average fade duration are evaluated. The second order statistics of MACD reception in the presence of fading is considered in paper [11]. Average fade duration and level crossing rate of selection combining receiver are evaluated in paper [12].

In this paper, MACD technique with MACD SC receiver and two MICD MRC receivers where in the first MRC receiver is presented Nakagami- $m$  short term fading and in the second MRC receiver is presented  $\kappa$ - $\mu$  multipath fading is considered and closed form expression for level crossing rate at SC receiver output signal envelope is calculated. To the best of author's knowledge MACD system in the presence of mixed Nakagami- $m$  short term fading and  $\kappa$ - $\mu$  short term fading is not reported in open technical literature. Obtained results can be used in designing and performance analyzes of MACD reception in the presence of long term fading and short term fading.

## II. LEVEL CROSSING RATE AT OUTPUTS OF MRC RECEIVERS

Macrodiversity system considered in this paper, has macrodiversity SC receiver and two microdiversity MRC receivers. Received signal at the first MRC receiver is subjected to Nakagami- $m$  short term fading and Gamma long term fading and receiver signal at the second MRC receiver is subjected to  $\kappa$ - $\mu$  short term fading and Gamma long term fading. Model of considered macrodiversity system is shown in Fig. 1.

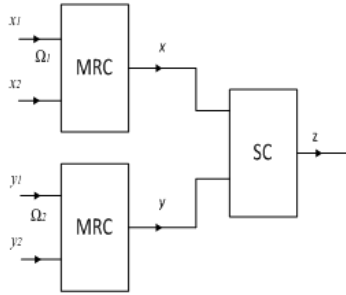


Figure 1. Model of proposed systems

Signal envelopes at inputs and outputs of microdiversity MRC receivers and signal envelopes at inputs and output of macrodiversity SC receiver are denoted as shown at Figure 1. Signal envelopes  $x_1$  and  $x_2$  follow Nakagami- $m$  distribution:

$$p_{x_i}(x_i) = \frac{2}{\Gamma(m)} \left( \frac{m}{\Omega_1} \right)^m x_i^{2m-1} e^{-\frac{m}{\Omega_1} x_i^2}, \quad x_i \geq 0, \quad i=1,2, \quad (1)$$

and signal envelope  $y_1$  and  $y_2$  follow  $\kappa$ - $\mu$  distribution:

$$\begin{aligned} p_{y_i}(y_i) &= \frac{2\mu(\kappa+1)^{\frac{\mu+1}{2}} y_i^{\mu}}{\kappa^{\frac{\mu-1}{2}} e^{\kappa\mu} \Omega_2^{\frac{\mu+1}{2}}} e^{-\frac{\mu(\kappa+1)}{\Omega_2} y_i^2} I_{\mu-1} \left( 2\mu \sqrt{\frac{\kappa(\kappa+1)}{\Omega_2}} y_i \right) = \\ &= \frac{2\mu(\kappa+1)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} e^{\kappa\mu} \Omega_2^{\frac{\mu+1}{2}}} \sum_{i=0}^{\infty} \left( 2\mu \sqrt{\frac{\kappa(\kappa+1)}{\Omega_2}} \right)^{2i+\mu-1} \frac{1}{i_1! \Gamma(i_1 + \mu)} \\ &\times y_i^{2i+2\mu-1} e^{-\frac{\mu(\kappa+1)}{\Omega_2} y_i^2}, \quad x_i \geq 0, \quad i=1,2. \end{aligned} \quad (2)$$

In previous expressions parameter  $m$  is Nakagami- $m$  fading severity parameter,  $\Omega_1$  is average power of Nakagami- $m$  short term fading,  $\kappa$  is Rician factor of  $\kappa$ - $\mu$  short term fading,  $\mu$  is severity parameter of  $\kappa$ - $\mu$  multipath fading and  $\Omega_2$  is average power of  $\kappa$ - $\mu$  small scale fading. Joint probability density function of signal envelope at output of the first microdiversity MRC receivers and its first derivative is:

$$p_{x\dot{x}}(x\dot{x}) = \frac{2}{\Gamma(2m)} \left( \frac{m}{\Omega_1} \right)^{2m} x^{4m-1} e^{-\frac{m}{\Omega_1} x^2} \frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}} e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}}, \quad x_i \geq 0, \quad (3)$$

where:

$$\sigma_{\dot{x}}^2 = \pi^2 f_m^2 \frac{\Omega_1}{m}, \quad (4)$$

$f_m$  is maximal Doppler frequency. Level crossing rate of random process of signal envelope  $x$  is [13]:

$$\begin{aligned} N_x &= \int_0^{\infty} d\dot{x} \dot{x} p_{x\dot{x}}(x\dot{x}) = \frac{2}{\Gamma(2m)} \left( \frac{m}{\Omega_1} \right)^{2m} x^{4m-1} e^{-\frac{m}{\Omega_1} x^2} \frac{1}{\sqrt{2\pi}} \sigma_{\dot{x}} = \\ &= \frac{\sqrt{2\pi} f_m}{\Gamma(2m)} \frac{m^{2m-1/2}}{\Omega_1^{2m-1/2}} x^{4m-1} e^{-\frac{m}{\Omega_1} x^2}. \end{aligned} \quad (5)$$

The joint probability density function of signal envelope process at output of the second MRC receiver and its first derivative is:

$$\begin{aligned} p_{y\dot{y}}(y\dot{y}) &= \frac{4\mu(\kappa+1)^{\frac{2\mu+1}{2}}}{\kappa^{\frac{2\mu-1}{2}} e^{2\kappa\mu} 2\Omega_2^{\frac{\mu+1}{2}}} \sum_{i=0}^{\infty} \left( 2\mu \sqrt{\frac{\kappa(\kappa+1)}{2\Omega_2}} \right)^{2i+2\mu-1} \\ &\times \frac{1}{i_1! \Gamma(i_1 + 2\mu)} y^{2i+4\mu-1} e^{-\frac{\mu(\kappa+1)}{\Omega_2} y^2} \frac{1}{\sqrt{2\pi}\sigma_{\dot{y}}} e^{-\frac{\dot{y}^2}{2\sigma_{\dot{y}}^2}}, \end{aligned} \quad (6)$$

$$\text{where: } \sigma_{\dot{y}}^2 = \pi^2 f_m^2 \frac{\Omega_2}{\mu(\kappa+1)}.$$

Level crossing rate of  $y$  is [13]:

$$\begin{aligned} N_x &= \int_0^{\infty} d\dot{y} \dot{y} p_{y\dot{y}}(y\dot{y}) = p_y(y) \frac{\sigma_{\dot{y}}}{\sqrt{2\pi}} = \\ &= \frac{2\mu^{1/2} (\kappa+1)^{\mu} \sqrt{2\pi} f_m}{\kappa^{\mu-1/2} e^{2\kappa\mu} 2^{\frac{2\mu+1}{2}} \Omega_2^{\mu}} \sum_{i=0}^{\infty} \left( 2\mu \sqrt{\frac{\kappa(\kappa+1)}{2\Omega_2}} \right)^{2i+2\mu-1} \\ &\times \frac{1}{i_1! \Gamma(i_1 + 2\mu)} y^{2i+4\mu-1} e^{-\frac{\mu(\kappa+1)}{\Omega_2} y^2}. \end{aligned} \quad (7)$$

Obtained expression for level crossing rate (8), can be used for calculation average fade duration of wireless communication system with MRC diversity receiver with two inputs operating over  $\kappa$ - $\mu$  small scale fading channel.

Derived expression for level crossing rate (5), can be used for evaluated the second order performance measures of wireless mobile communication radio system with MRC diversity receiver the presence Nakagami- $m$  multipath fading.

### III. LEVEL CROSSING RATE OF SC RECEIVER OUTPUT SIGNAL ENVELOPE

Signal envelopes average powers at inputs of microdiversity MRC receivers  $\Omega_1$  and  $\Omega_2$  follow correlated Gamma distribution:

$$p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) = \frac{1}{\Gamma(c)(1-\rho^2)\rho^{c-1}\Omega_0^{c+1}} \sum_{i_2=0}^{\infty} \left( \frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2i_2+c-1} \times \frac{1}{i_2!\Gamma(i_2+c)} \Omega_1^{i_2+c-1} \Omega_2^{2i_2+c-1} e^{-\frac{\Omega_1+\Omega_2}{\Omega_0(1-\rho^2)}} \quad (8)$$

Macrodiversity SC receiver selects the first microdiversity MRC receiver to provide service to user when signal envelope average power at its inputs is higher of signal envelope average power at of inputs of the second microdiversity MRC receiver. Also, macrodiversity SC receiver selects the second microdiversity receiver to enable service to user when signal envelope average power at its inputs is higher than signal envelope average power at inputs of the first MRC receiver. Therefore, level crossing rate at output of SC receiver is:

$$N_z = \int_0^{\infty} d\Omega_1 \int_0^{\Omega_1} d\Omega_2 N_x / \Omega_1 p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) + \int_0^{\infty} d\Omega_2 \int_0^{\Omega_2} d\Omega_1 \times N_y / \Omega_2 p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) = \sqrt{2\pi} f_m m^{2m-1/2} z^{4m-1} \times \frac{1}{\Gamma(c)(1-\rho^2)\rho^{c-1}\Omega_0^{c+1}} \sum_{i_2=0}^{\infty} \left( \frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2i_2+c-1} \frac{1}{i_2+c} \times \frac{1}{i_2!\Gamma(i_2+c)} \sum_{j_1=0}^{\infty} \frac{1}{(i_2+c+j_1)j_1} \left( \frac{1}{\Omega_0(1-\rho^2)} \right)^{j_1} \times \left( \frac{mz^2\Omega_0(1-\rho^2)}{2} \right)^{i_2+c+j_1/2-m+1/4} \times K_{2i_2+2c+j_1-2m+1/2} \left( 2\sqrt{\frac{2mz^2}{\Omega_0(1-\rho^2)}} \right) + \frac{2\mu^{1/2}(\kappa+1)^\mu \sqrt{2\pi} f_m}{\kappa^{\mu-1/2} e^{2\kappa\mu} 2^{m+1/2}} \sum_{i_1=0}^{\infty} \left( 2\mu\sqrt{\frac{\kappa(\kappa+1)}{2\Omega_2}} \right)^{2i_1+2\mu-1}$$

$$\times \frac{1}{i_1!\Gamma(i_1+2\mu)} z^{2i_1+4\mu-1} \frac{1}{\Gamma(c)(1-\rho^2)\rho^{c-1}\Omega_0^{c+1}} \times \sum_{i_2=0}^{\infty} \left( \frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2i_2+c-1} \frac{1}{i_2!\Gamma(i_2+c)} \frac{1}{i_2+c} \quad (5) \times \sum_{j_2=0}^{\infty} \frac{1}{(i_2+c+j_2)j_2} \left( \frac{\mu(\kappa+1)z^2\Omega_0(1-\rho^2)}{2} \right)^{i_2+c+j_2/2-m-i_1/2+1/4} \times \left( \frac{1}{\Omega_0(1-\rho^2)} \right)^{j_2} K_{2i_2+2c+j_2-2m-2i_1+1/2} \left( 2\sqrt{\frac{2\mu(\kappa+1)z^2}{\Omega_0(1-\rho^2)}} \right)$$

### IV. NUMERICAL RESULTS

In numerical results, LCRs normalized by  $f_m$  are presented on figures 2-3. Figure 2. shows LCR versus signal envelope for constant values of parameters of Nakagami- $m$  fading severity,  $\kappa$ - $\mu$  fading severity, shadowing severity, Rician factor and various values of  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_0$ . It is evident that LCR increases for lower values of  $z$ , reaches its maximum, than for higher values of  $z$  LCR decreases.

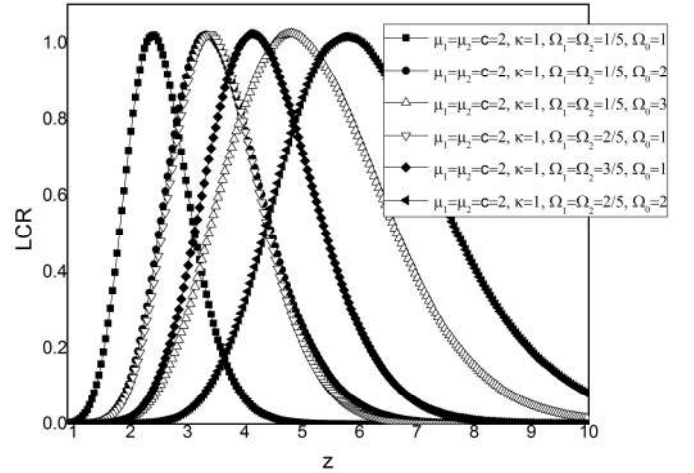


Figure 2. LCR for various parameters values  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_0$  constant values of  $\mu_1$ ,  $\mu_2$ ,  $c$ , and  $\kappa$

Figure 3 presents LCR versus  $z$  for various values of Rician factor and constant values of other system model parameters. When parameter  $\kappa$  increases, LCR maximums moves in the region with the higher values of  $z$ .

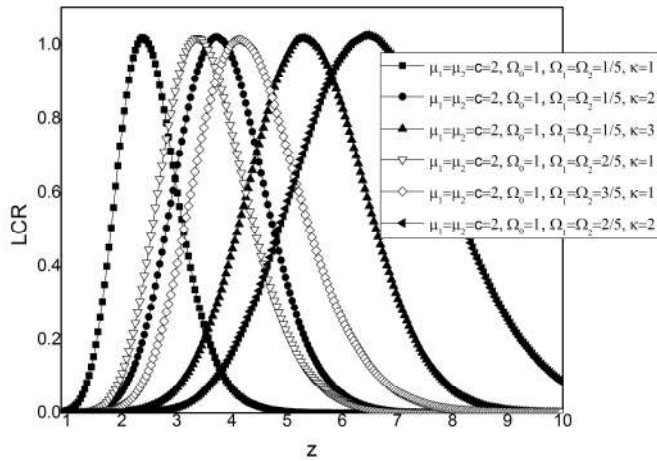


Figure 3. LCR for various parameters values  $\Omega_1$ ,  $\Omega_2$  and  $\kappa$  constant values of  $\mu_1$ ,  $\mu_2$ ,  $c$ , and  $\Omega_0$

## V. CONCLUSION

MACD system with MACD SC receiver and two MRC MICD receivers operating over shadowed multipath fading channel is analyzed. Received signal in the first MACD MRC receiver is subjected to Nakagami- $m$  short term fading. Nakagami- $m$  distribution describes signal envelope in channel with no dominant component and  $\kappa$ - $\mu$  distribution describes signal envelope in channel with line-of-sight dominant component. MACD reception reduces long term fading effects and short term fading effects on system performance simultaneously. MACD SC receiver selects MICD MRC receiver with higher signal envelope average power to provide service to user resulting in Gamma long term fading reduction. The first MICD MRC receiver selects the branch with the highest signal to interference ratio resulting in Nakagami- $m$  short term fading effects reduction and the second MICD MRC receiver selects to branch with the highest signal to interference ratio resulting in  $\kappa$ - $\mu$  short term fading effects reduction on system performances. In this paper, level crossing rate of SC receiver output signal envelope is calculated as closed form expression. The influence of Nakagami- $m$  short term fading severity parameter, Rician factor of  $\kappa$ - $\mu$  short term fading and Gamma long term fading severity parameter on level crossing rate is analyzed. Derived expression for level crossing rate can be used for evaluation of level crossing rate of MACD system in the presence Gamma long term fading, Rayleigh short term fading and Rician short term fading.

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