BER evaluation of PSK receiver over Weibull fading in the presence of imperfect carrier phase estimation

Jelena Anastasov, Nenad Milošević,
University of Nis, Faculty of Electronic Engineering
Serbia
jelena.anastasov@elfak.ni.ac.rs,
nenad.milosevic@elfak.ni.ac.rs

Zvezdan Marjanović, Goran T. Đorđević
University of Nis, Faculty of Electronic Engineering
Serbia
zvezdan.marjanovic@elfak.ni.ac.rs, goran@elfak.ni.ac.rs

Abstract— **In this paper, the error performance is analysed in reception of binary phase-shift keying (BPSK) and quaternary phase-shift keying (QPSK) signals transmitted over a multipath Weibull fading channel under assumption that reference carrier signal phase estimation is imperfect. Taylor series expansion based expressions for the average bit error rate (ABER) of BPSK and QPSK signal detection are derived. Obtained analytical results are in a form of Meijer's *G* functions computable in several mathematical software packages. The analytical expressions are derived in order to make possible efficient estimation of carrier phase error (CPE) effects on ABER performance. Simplification of Weibull fading scenario into Rayleigh fading case of proposed system is also given. Finally, in order to estimate the CPE degradation of ABER and to confirm the accuracy of given approximation results, numerical results are obtained, presented and commented.**

Key words – carrier phase error; coherent modulation; error performance; Meijer's G function; Taylor series expansion; Weibull fading

I. INTRODUCTION

In wireless communications, receivers with non-coherent detection are simpler to implement compared to the coherent detection scenarios, but those with coherent detection offer lower bit error rates. On the other hand implementation of coherent receivers requires perfect phase reference [1]. In real practical scenarios, the carrier phase reference is reconstructed from corrupted version of a received signal. So, the case of nonideal coherent detection refers to the fact of carrier phase error (CPE) appearance [2]. The estimated phase becomes imperfect mainly due to inherent noise in the phase recovery loop. Tikhonov and Viterbi introduced the model of the probability density function (pdf) of the phase error in first-order phase-locked loop (PLL), which was later shown that is also appropriate as a model for second order PLLs [1]-[3].

There are a numerous papers in the literature on the above topic. At the beginning, the imperfect carrier synchronization was treated over additive white Gaussian noise (AWGN) channels [3], and later on the analysis was extended to the fading corrupted systems [4]-[10]. The performance of such imperfect, namely partially coherent systems over Rayleigh, Rice and lognormal fading channels was presented in [4]. Involving diversity reception in the presence of the CPE, the authors in [5] and [6] have shown various performance evaluations of coherent phase-shift keying (PSK) systems under different fading environments. For example, in [5] maximal ratio combining and equal-gain combining (EGC) were obtained in tracking effects of imperfect carrier reception of binary PSK (BPSK) and quaternary (QPSK) in the presence of multipath fading. Based on Hermite polynomials expansion, an infinite series expression of the average bit error rate (ABER) under EGC diversity reception in Rayleigh fading channel was presented in [6]. The detection loss suffered by the carrier recovery for various signal-to-noise ratios (SNRs) reliability again with EGC receiver over generalized fading channels was investigated in [7].

In [8], Simon and Alouini defined simplified noisy reference loss formulae of partially coherent BPSK and QPSK systems in slow Nakagami-*m* fading channels. The reception of BPSK and QPSK signals over Gamma shadowed Rayleigh fading channel in the presence of phase noise was studied in [9]. For the first time, the ABER and outage probability of multibranch EGC receiver with CPE in the presence of co-channel interference were considered in [10].

In this paper, we analyze partially coherent systems over Weibull fading channels. Based on Taylor series expansion of the complementary error function (erfc) in ABER definition formulae for BPSK and QPSK formats, the error performance evaluation of proposed systems is obtained. New expressions in a form of Meijer's *G* function for evaluating ABER of aforementioned systems with Tikhonov distributed CPE are derived. In order to predict degradation introduced by CPE and to show fading severity influence, numerical results are also obtained. Accuracy of presented results is confirmed comparing derived approximated ABER results with the results obtained by performing numerical integrations.

II. ANALYSIS BACKGROUND

In our analysis, we assume transmission of signals over Weibull fading environment. The Weibull fading model describes a wide range of different fading severity conditions where the Rayleigh fading model is unadequate. View to the fact that it specializes to the Rayleigh fading model, the Weibull model is valid for both weak and strong fading severity scenarios [11].

The pdf of the anvelope r, modeled by Weibull distribution, has the following form [11]

$$p_r(r)=\frac{\beta}{\Omega}r^{\beta-1}\exp\left(-\frac{r^\beta}{\Omega}\right), \quad (1)$$

where $\Omega=\mathrm{E}\left[r^\beta\right]$ and β is the fading parameter (β >0). Smaller values of β parameter indicate severe fading conditions and vice versa.

In order to analyze the performance of a partially coherent communication system, we require the PDF of the instantaneous signal-to-noise ratio (SNR), γ_b, at the input of the receiver. This instantaneous SNR can be defined as $\gamma_b = r^2E_b/N_0$, so after simple transformation of (1), the PDF of γ_b is found as

$$p_{\gamma_b}(\gamma_b)=\frac{\beta}{2a\bar{\gamma}}\left(\frac{\gamma_b}{a\bar{\gamma}}\right)^{\beta/2-1}\exp\left(-\frac{\gamma_b}{a\bar{\gamma}}\right)^{\beta/2}, \quad (2)$$

where $\bar{\gamma}$ is the average SNR per bit and $a=\frac{1}{\Gamma(1+2/\beta)E_b/N_0}$, with E_b/N_0 being the energy-per-bit to noise power spectral density ratio.

We assume nonideal coherent detection with the CPE described by Tikhonov distribution. The PLL SNR, ρ, is defined as $\rho=P_c/(N_0B_L)$ with P_c representing the power needed for the carrier phase recovery pilot and B_L is the loop bandwidth. The part of the total power P_t is dedicated to the pilot in a way $P_c=\eta P_t$ where η is the fixed fraction of the P_t. The remaining fraction is reserved for data detection. All of this applies to $(1-\eta)P_t=E_b/T_b$, where T_b defines the bit interval. Thus, in specific fading channel, it is easy to obtain the PLL SNR as [8]

$$\rho=\frac{\eta}{1-\eta}\frac{1}{B_LT_b}\frac{E_b}{N_0}. \quad (3)$$

The previous equation shows that the PLL SNR is proportional to the instantaneous received SNR and can be rewritten as, $\rho=C\gamma_b$, where C is a constant defined as $C=\eta/\left((1-\eta)B_LT_b\right)$. Parameter C represents the amount by which the loop SNR (in dB) exceeds the energy-per-bit to noise power spectral density ratio (in dB).

Let $\varphi=\theta-\hat{\theta}$ denotes the phase error (θ is the phase of the received carrier and $\hat{\theta}$ is nonideal estimation of the phase of carrier reference provided at the receiver), so the PDF of the CPE, φ, has the following form [3]

$$p_\varphi(\varphi|\gamma_b)=\frac{e^{C\gamma_b\cos\varphi}}{2\pi\mathrm{I}_0(C\gamma_b)}, \quad -\pi\le\theta\le\pi, \quad (4)$$

with $\mathrm{I}_0(.)$ denoting the first kind modified Bessel function of order zero [12].

III. AVERAGE BIT ERROR RATE EVALUATION

In the case of imperfect coherent BPSK detection, the conditional ABER can be evaluated as [8]

$$P_b^{BPSK}(\varphi,\gamma_b)=\frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma_b}\cos\varphi\right), \quad (5)$$

where $\mathrm{erfc}(x)=\left(2/\sqrt{\pi}\right)\int_x^\infty e^{-t^2}dt$ is the complementary error function [12]. By using classic probability theory analysis, the ABER for any modulation format can be obtained as

$$P_b=\int_0^{+\infty}\int_{-\pi}^{\pi}P_b(\varphi,\gamma_b)p_\varphi(\varphi|\gamma_b)p_{\gamma_b}(\gamma_b)d\varphi d\gamma_b, \quad (6)$$

where $P_b(\varphi,\gamma_b)$ presents the conditional ABER for required modulation format.

Substituting (2), (4) and (5) into (6), exact values of the ABER can be evaluated numerically because an exact closed-form solution of two-fold integral in (6) does not exist. But if we adopt the expansion of erfc function into a Taylor series as [13]

$$P_{b/\gamma_b}^{BPSK}\cong\frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma_b}\right)+\frac{1}{2}\sigma_\varphi^2\sqrt{\frac{\gamma_b}{\pi}}e^{-\gamma_b}+\text{higher order terms}, \quad (7)$$

approximation of ABER for imperfect detection can be obtained by averaging the first two terms of (7) over the PDF $p_{\gamma_b}(\gamma_b)$, as

$$P_{noideal}^{BPSK}\cong\int_0^{+\infty}\left(\frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma_b}\right)+\frac{1}{2}\sigma_\varphi^2\sqrt{\frac{\gamma_b}{\pi}}e^{-\gamma_b}\right)p_{\gamma_b}(\gamma_b)d\gamma_b, \quad (8)$$

with σ_φ^2 being CPE variance. In practice, the conditions are often such that $\rho >> 1$, so it is reasonable to adopt the approximation of CPE variance as $\sigma_\varphi^2\approx 1/\rho$.

Now, substituting (2) in (8) and transforming erfc and exp function into Meijer's G functions using [14, eq. (8.4.14.2)] and [14, eq. (8.4.3.1)] respectively, both integrands can be solved by applying the procedure of integrating product of two Meijer's G functions [14, eq. (2.24.1.1)] in the way

$$P_{noideal}^{BPSK} = \frac{1}{4}\sqrt{\frac{2}{\pi}}\left(\frac{\beta}{2\pi a\bar{\gamma}}\right)^{\frac{\beta}{2}} G_{2\beta,2+\beta}^{2,2\beta}\left(\left(\frac{\beta}{a\bar{\gamma}}\right)^{\beta}\frac{1}{4}\middle|\begin{matrix}\Delta_{(\beta)}^{(\beta/2)},\Delta_{(\beta)}^{((\beta+1)/2)}\\ 0,\frac{1}{2},\Delta_{(\beta)}^{(\beta/2+1)}\end{matrix}\right)$$

$$+\frac{1}{4C}\sqrt{\frac{2}{\pi}}\left(\frac{\beta}{2\pi a\bar{\gamma}}\right)^{\frac{\beta}{2}} G_{\beta,2}^{2,\beta}\left(\left(\frac{\beta}{a\bar{\gamma}}\right)^{\beta}\frac{1}{4}\middle|\begin{matrix}\Delta_{(\beta)}^{((\beta-1)/2)}\\ 0,\frac{1}{2}\end{matrix}\right). \quad (9)$$

with $G_{p,q}^{m,n}\left(z\middle|\begin{matrix}(t_i)_{1,p}\\(k_j)_{1,q}\end{matrix}\right)$ denoting Meijer's G function [12, eq. (9.301)], $\Delta_{(c)}^{(b)} = \frac{1-b}{c},\frac{2-b}{c},\cdots,\frac{c-b}{c}$, for $c\in N$ and $b\in N$, when fading severity parameter $\beta\in N$.

For $\beta = 2$, the previous formula can be simplified to the case of Rayleigh fading scenario in the way

$$P_{noideal,\text{Reyleigh}}^{BPSK} = \frac{1}{4}\sqrt{\frac{2}{\pi}}\frac{1}{\pi a\bar{\gamma}} G_{4,4}^{2,4}\left(\frac{1}{(a\bar{\gamma})^2}\middle|\begin{matrix}0,\frac{1}{2},-\frac{1}{4},\frac{1}{4}\\ 0,\frac{1}{2},-\frac{1}{2},0\end{matrix}\right)$$

$$+\frac{1}{4C}\sqrt{\frac{2}{\pi}}\frac{1}{\pi a\bar{\gamma}} G_{2,2}^{2,2}\left(\frac{1}{(a\bar{\gamma})^2}\middle|\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{1}{2}\end{matrix}\right). \quad (10)$$

Using transformation of Meijer's G function into more familiar hypergeometric functions [14, eq. (8.2.2.3)], and after adequate mathematical manipulation, (10) is further simplified into the ABER approximation formula for Rayleigh fading scenario presented in [8, eq. (11)], in the following way

$$P_{noideal,\text{Reyleigh}}^{BPSK} = \frac{1}{2}\left[1-\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right]+\frac{1}{2C\sqrt{\bar{\gamma}}(1+\bar{\gamma})}. \quad (11)$$

When only the first integrand of (8) is considered, the perfect (ideal) carrier phase reference ABER can be evaluated as

$$P_{ideal}^{BPSK} = \frac{1}{4}\sqrt{\frac{2}{\pi}}\left(\frac{\beta}{2\pi a\bar{\gamma}}\right)^{\frac{\beta}{2}} G_{2\beta,2+\beta}^{2,2\beta}\left(\left(\frac{\beta}{a\bar{\gamma}}\right)^{\beta}\frac{1}{4}\middle|\begin{matrix}\Delta_{(\beta)}^{(\beta/2)},\Delta_{(\beta)}^{((\beta+1)/2)}\\ 0,\frac{1}{2},\Delta_{(\beta)}^{(\beta/2+1)}\end{matrix}\right). \quad (12)$$

The above results can be further extended to the QPSK modulation case according to the Taylor series expansion of the conditional QPSK ABER formulated as [8]

$$P_b^{QPSK}(\varphi,\gamma_b) = \frac{1}{4}erfc\left(\sqrt{\gamma_b}(\cos\varphi+\sin\varphi)\right)+\frac{1}{4}erfc\left(\sqrt{\gamma_b}(\cos\varphi-\sin\varphi)\right) \quad (13)$$

Analogously to the series expansion of conditional BPSK ABER, (13) becomes

$$P_{b/\gamma}^{QPSK} \cong \frac{1}{2}\text{erfc}\left(\sqrt{\gamma_b}\right)+\frac{1}{2}\sigma_\varphi^2\sqrt{\frac{\gamma_b}{\pi}}(1+2\gamma_b)e^{-\gamma_b} + \text{higher order terms}. \quad (14)$$

Neglecting higher order terms of (14) and following the same procedure as in solving (9), the ABER of imperfect detection for QPSK is derived in the following way

$$P_{noideal}^{QPSK} = P_{noideal}^{BPSK} + \frac{\beta}{2C}\sqrt{\frac{2}{\pi}}\left(\frac{\beta}{2\pi a\bar{\gamma}}\right)^{\frac{\beta}{2}} G_{\beta,2}^{2,\beta}\left(\left(\frac{\beta}{a\bar{\gamma}}\right)^{\beta}\frac{1}{4}\middle|\begin{matrix}\Delta_{(\beta)}^{((\beta+1)/2)}\\ 0,\frac{1}{2}\end{matrix}\right). \quad (15)$$

As expected, an additional degradation of ABER for QPSK is noticable comparing (9) with (15). ABER when (ideal) carrier phase reference exist can be evaluated adopting $P_{ideal}^{QPSK} = P_{ideal}^{BPSK}$.

IV. Numerical Results

Fig. 1. and Fig. 2. show ABER dependence on the average SNR of partially coherent system for BPSK and QPSK modulation schemes, respectively. As expected, the increase of fading parameter β brings an improvement of ABER for both modulation schemes. Excellent agreement of approximations and numerical results obtained by numerical integration is noticeable in the range from weak to strong severity fading conditions for BPSK modulation format. From Fig. 2., in the case of QPSK modulation scheme, we notice small deviation of ABER values comparing exact and approximated analytical results for higher values of β (e.g. $\beta = 6$), but still the accuracy of utilized approximation approach is obvious.

From the Fig. 1., negligible values of deviation between ABER in the case of ideal and imperfect coherent detection for any SNR value can be observed, which claims redundancy of phase synchronization in the case of BPSK. Opposite to this, higher values of this deviation, when QPSK is applied, can be noticed. So, the phase compensation in the case of QPSK is contested. For instance, for QPSK modulation scheme at the ABER level of 10^{-6} and $\beta = 6$, this deviation in SNR is about 1.5 dB, unlike the BPSK modulation case when this value is only 0.1 dB for the same specified parameters. Curves for $\beta = 2$ refer to the case of Rayleigh fading scenario.

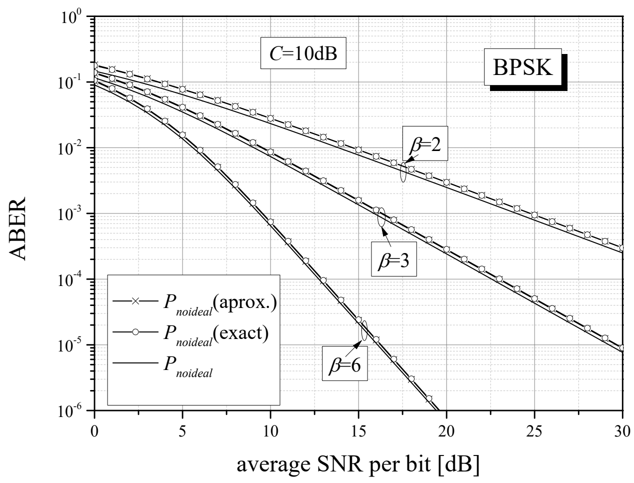

Fig 1. ABER of partially coherent system with BPSK modulation scheme under different fading conditions

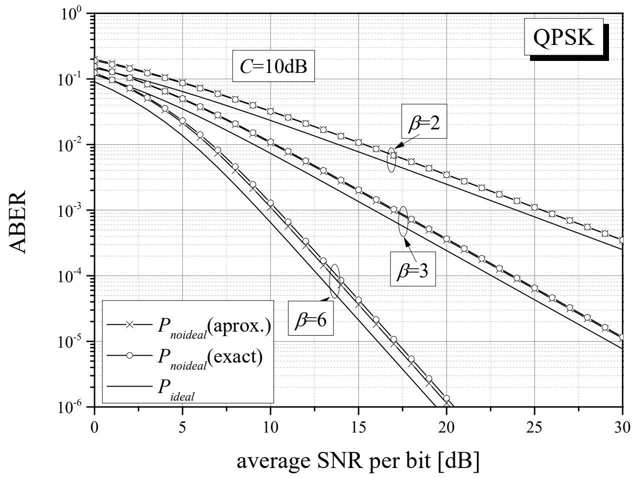

Fig 2. ABER of partially coherent system with QPSK modulation scheme over Weibull fading channel

Fig. 3. Illustrates the ABER performance of proposed system with CPE and QPSK for different values of loop parameter *C*. When the PLL parameter is above approximately 8 dB, which are the typical values of *C* in practice, the derived expressions can be used safely as quite accrue. For higher values of PLL parameter, the ABER performance improvement exists. When we compare curves for $C = 10$ dB and $C = 15$ dB the improvement in ABER is larger than the improvement in ABER when *C* increase from $C = 15$ dB to $C = 20$ dB over the wide range of fading severity. For instance, the penalty in SNR values to reach ABER of 10^{-5} when *C* changes from 15 dB to 10 dB is 1 dB, while the penalty in SNR when *C* changes from 20 dB to 15 dB equals 0.2 dB, for $\beta = 5$. This gap is more obvious in the case of weaker fading severity.

Finally, we can notice that the case with severe fading condition ($\beta = 1$), deeper than the Rayleigh fading condition, is unacceptable for transmission over the range of average SNR up to 30 dB.

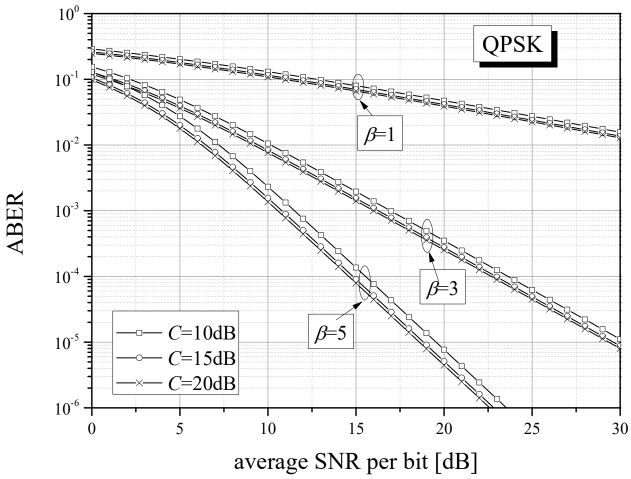

Fig 3. ABER of partially coherent system with QPSK modulation scheme for different values of PLL parameter

V. Conclusion

In this work, error performance evaluation of partially coherent systems with Tikhonov distributed CPE under Weibull fading conditions has been presented. Based on Taylor series expansion for conditional ABER of BPSK and QPSK, new approximation results of system ABER have been derived. There is an excellent agreement between the approximations and the results obtained by performing numerical integration, confirming the accuracy of the proposed approach. Besides, obtained analytical results in a form of Meijer's *G* functions need much shorter time of computing compared with the two-fold numerical integration results computation under controlled accuracy. Our analysis has also shown that unlike BPSK, QPSK modulation scheme is more sensitive to phase estimation error. The mathematical derivations presented here allow us to estimate performance when fading severity parameter is integer. For non-integer values it is necessary to make interpolation. Our further focus will be to develop efficient mathematical framework, which will enable us to estimate ABER for any value of Weibull severity parameter. This analysis will be based on application of Fox's H functions.

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