Analytical Derivation of Moments of System Applying Diversity at Micro and Macro Level in Gamma Shadowed Nakagami-*m* Fading Channels

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Abstract— **In this paper, we derive a closed-form expression for moments of system with maximal-ratio combining (MRC) microdiversity and selection combining (SC) macrodiversity reception operating in correlated gamma shadowed Nakagami-*m* fading channel, and in this way finalize our previous works related to investigation of first- and second-order performance measures of described system. In addition, we focus on research of influence of system and channel parameters on both average output signal and amount of fading (AoF).**

Keywords-fading; macrodiversity; microdiversity; shadowing; system moment

I. INTRODUCTION

In wireless communication, two independent propagation phenomena can be distinguished: fading and shadowing. Fading is fluctuation in amplitude of received signal caused by multipath components which occur due to scattering, diffraction and reflection. Shadowing is effect that received signal power fluctuates due to obstacles obstructing the propagation between transmitter and receiver. In composite fading environment, in which both propagation phenomena influence on performance degradation, a spatial diversity at micro and macro level should be applied to mitigate that degradation. A base station (BS) equipped with multiple antennas determines microdiversity, while process of signals originating from several BSs represents macrodiversity. Among well known spatial diversity techniques, maximal-ratio combining (MRC) gives the best system performance, while selection combining (SC) provides the least implementation complexity. Therefore, SC technique is recommended to be used at macro level, and in order to achieve the best possible system performance, MRC technique should be used at micro level.

In open technical literature, Rayleigh, Rician, Weibull and Nakagami-*m* distributions are the most frequently used for characterization of signal envelope variation. In this paper, Nakagami-*m* statistical model is used to describe multipath fading, because this model gives a possibility to cover a wide range of the multipath fading by adjusting fading parameter m [1-2]. Shadowing is often modeled with lognormal distribution [3-5]. However, that distribution does not allow expressing probability density function (PDF) of received signal within a closed-form expression making system analysis very ponderous. On the other hand, papers [6-10] confirm, trough both theoretical and measured results, that lognormal and gamma distributions match very well, and moreover, gamma model for describing average signal power variations leads to solution for the signal PDF in closed-form.

In papers [11-12], the system with macro diversity that is based on selection the best BS applying MRC technique at the micro level in a composite multipath/shadowed fading environment modelled as Nakagami/gamma is studied. Actually, paper [12] studies first-order performance measures (outage probability, channel capacity, bit error probability), while in [11] authors investigate second-order performance measures (average level crossing rate and average fade duration). This paper finalizes the work done in [11-12] deriving the closed-form expression for the n-th moment of signal at the output of previously described system.

II. CHANNEL AND SYSTEM MODEL

Considered wireless communication system consists of microdiversity and macrodiversity applied at the reception side. Namely, N BSs are located at different geographical positions in the same cell and BS which services user is selected by SC algorithm. BSs are equipped with L antennas and signals from antennas are combined using MRC algorithm. The described system is presented in Fig. 1.

To provide uncorrelated signals at BS's antennas, the distance between them has to be on the order of one half of a wavelength [13] and often this condition is fulfilled. On the other hand, BSs are usually shadowed by same obstacles causing space correlation between them [14]. Therefore, the PDF of Nakagami-*m* signal envelope r_{ij} received by i-th antenna at j-th BS is given by [1, equation (3)]

$$f_{r_{ij}}\left(r_{ij}\right)=\frac{2m^{m}r_{ij}^{2m-1}}{\Gamma\left(m\right)\Omega_{j}^{\,m}}\exp\left(-\frac{m}{\Omega_{j}}r_{ij}^{2}\right),\ i=\overline{1,L}\,,\ j=\overline{1,N}, \quad (1)$$

where Ω_j represents the average power of signal at the j-th BS, m presents Nakagami fading parameter describing fading severity ($m \geq 0.5$) and $\Gamma(\cdot)$ is gamma function.

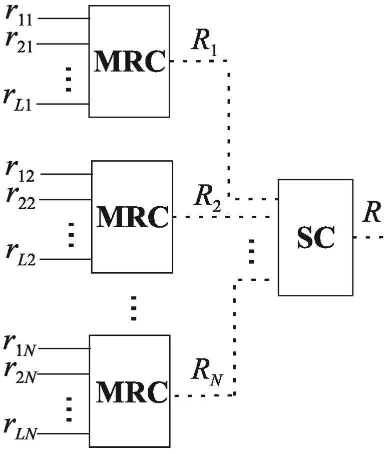

Figure 1. Wireless communications system model with multibranch microdiversity and macrodiversity reception

It is well-known that the result signal at the receiver output of the j-th BS employing MRC algorithm can be derived as the sum of squared envelopes of Nakagami-m faded signals, $R_j = \sum_{i=1}^{L} r_{ij}^2$, with the PDF [15]

$$f_{R_j}\left(R_j \middle| y_j\right) = \frac{R_j^{M-1} M^M}{\Gamma(M)\, y_j^M} \exp\left(-\frac{M}{y_j} R_j\right), \; j = \overline{1,N}, \tag{2}$$

where $M = Lm$ and y_j is the input average power in total ($y_j = L\Omega_j$). The conditioning in (2) demonstrates the presence of shadowing where y_j is random variable. Considering the exponential correlation and identical distribution of random variables y_j, the joint PDF of total input average powers is [16, equation (13) for $v = 0.5$]

$$f_{y_1 y_2 \ldots y_N}\left(y_1, y_2, \ldots, y_N\right) = \frac{\exp\left(-\dfrac{y_1 + y_N + (1+\rho)\sum_{i=2}^{N-1} y_i}{y_0(1-\rho)}\right)}{\rho^{\frac{c-1}{2}(N-1)} (1-\rho)^{N-1} {y_0}^{N+c-1} \Gamma(c)} \tag{3}$$
$$\times {y_1}^{\frac{c-1}{2}} {y_N}^{\frac{c-1}{2}} \prod_{i=1}^{N-1} I_{c-1}\left(\frac{2\sqrt{\rho}}{y_0(1-\rho)} \sqrt{y_i y_{i+1}}\right),$$

where ρ is the coefficient of correlation between random variables y_k and y_{k+1}, c is the order of gamma distribution, y_0 is related to the average power, and $I_n(\cdot)$ is the first kind and n-th order modified Bessel function. There is a connection between standard deviation σ of shadowing in dB in the lognormal shadowing and the parameter c through $\sigma(dB) = 4.3429\sqrt{\psi'(c)}$ [8, equation (17)], where $\psi'(\cdot)$ is the trigamma function. In practice, typical values of σ are between 2-12 dB.

When servicing user, BS with the largest total input average power is selected to serve it. So, after diversity combining at the micro and macro level, the signal PDF can be expressed as

$$\begin{aligned} f_R(R) = & \int_0^{\infty} dy_1 \int_0^{y_1} dy_2 \ldots \int_0^{y_1} f_{R_1}\left(R \middle| y_1\right) f_{y_1 y_2 \ldots y_N}\left(y_1, y_2, \ldots, y_N\right)\, dy_N \\ & + \int_0^{\infty} dy_2 \int_0^{y_2} dy_1 \ldots \int_0^{y_2} f_{R_2}\left(R \middle| y_2\right) f_{y_1 y_2 \ldots y_N}\left(y_1, y_2, \ldots, y_N\right)\, dy_N \\ & + \ldots + \int_0^{\infty} dy_N \int_0^{y_N} dy_1 \ldots \int_0^{y_N} f_{R_N}\left(R \middle| y_N\right) f_{y_1 y_2 \ldots y_N}\left(y_1, y_2, \ldots, y_N\right)\, dy_{N-1}, \end{aligned} \tag{4}$$

which by substituting (2) and (3) and after some mathematical manipulations and integrations using [17, equations (3.381/2) and (3.471/9)] yields [11]

$$\begin{aligned} f_R(R) = & \, 2\xi R^{\alpha_1 - 1} \\ & \times \sum_{n_1, n_2, \ldots, n_{N-1}=0}^{\infty} \; \sum_{k_1, k_2, \ldots, k_{N-1}=0}^{\infty} \frac{\rho^{\sum_{i=1}^{N-1} n_i} (1+\rho)^{\sum_{i=2}^{N-2} k_i}}{\prod_{i=1}^{N-1} \left(n_i! \Gamma(n_i + c)\right)} \\ & \times \left(\frac{MR}{\left(N + (N-2)\rho\right) y_0 (1-\rho)}\right)^{\beta} \frac{1}{\prod_{l_1=0}^{k_1} \left(n_{N-1} + c + l_1\right)} \\ & \times K_{Nc-M+\sum_{i=1}^{N-1}(2n_i + k_i)}\left(2\sqrt{\frac{MR\left(N + (N-2)\rho\right)}{y_0(1-\rho)}}\right) \\ & \times \frac{1}{\prod_{j=2}^{N-2} \prod_{l_j=0}^{k_j} \left(n_{N-j} + n_{N-j+1} + c + l_j\right)} \\ & \times \left(\frac{2(1+\rho)^{k_{N-1}}}{\prod_{l_{N-1}=0}^{k_{N-1}} \left(n_1 + n_2 + c + l_{N-1}\right)} + \frac{N-2}{\prod_{l_{N-1}=0}^{k_{N-1}} \left(n_1 + c + l_{N-1}\right)}\right), \end{aligned} \tag{5}$$

where

$$\xi = \frac{(1-\rho)^c}{\left(N + (N-2)\rho\right)^{\alpha_2} \Gamma(M) \Gamma(c)} \left(\frac{M}{y_0(1-\rho)}\right)^{\alpha_1}, \tag{6}$$
$$\alpha_1 = \frac{Nc+M}{2}, \; \alpha_2 = \frac{Nc-M}{2}, \; \beta = \sum_{i=1}^{N-1}\left(n_i + \frac{k_i}{2}\right),$$

and $K_n(\cdot)$ is the n-th order modified Bessel function of the second kind.

III. N-TH MOMENT OF SYSTEM

Since output signal is related to detection process, it is excellent indicator to the overall fidelity of the system [18]. Moreover, average output signal is the performance measure which is the easiest to be evaluated, because it requires knowledge of only the first statistical moment of instantaneous output signal.

One of the aims of diversity system is to reduce relative variance of the signal envelope. Therefore, it is necessary to take into account higher moments of output signal. Amount of fading (AoF) describes severity of fading channel and for its evaluation is necessary to know the second moment. Having in mind that AoF is evaluated at the output of diversity combiner, its evaluation shows not only statistics of fading channel, but behavior of the applied diversity technique.

Based on both previous explanation and knowledge that moment's order higher than two gives important shape about PDF of output signal, it is clear that evaluation of the n-th moment of output signal, R_n, is very important. It is defined as

$$R_n = \int_0^{+\infty} R^n f_R(R) dR. \tag{7}$$

Introducing (5) into (7) and after applying [17, equations (6.561/16)] to solve the integral, the closed form for R_n is presented as

$$\begin{aligned} R_n &= \xi \Gamma(M+n) \Gamma(Nc+n+2\beta) \\ &\times \sum_{n_1, n_2, \ldots, n_{N-1}=0}^{\infty} \sum_{k_1, k_2, \ldots, k_{N-1}=0}^{\infty} \frac{\rho^{\sum_{i=1}^{N-1} n_i} (1+\rho)^{\sum_{i=2}^{N-2} k_i}}{\prod_{i=1}^{N-1} \left(n_i! \Gamma(n_i+c)\right)} \\ &\times \left(\frac{M}{\left(N+(N-2)\rho\right) y_0 (1-\rho)} \right)^{\beta} \frac{1}{\prod_{l_1=0}^{k_1} \left(n_{N-1}+c+l_1\right)} \\ &\times \frac{1}{\prod_{j=2}^{N-2} \prod_{l_j=0}^{k_j} \left(n_{N-j}+n_{N-j+1}+c+l_j\right)} \\ &\times \left(\frac{2(1+\rho)^{k_{N-1}}}{\prod_{l_{N-1}=0}^{k_{N-1}} \left(n_1+n_2+c+l_{N-1}\right)} + \frac{N-2}{\prod_{l_{N-1}=0}^{k_{N-1}} \left(n_1+c+l_{N-1}\right)} \right). \end{aligned} \tag{8}$$

It is evident that average output signal is R_1, while AoF can be evaluated using

$$AoF = \frac{R_2}{R_1^2} - 1. \tag{9}$$

IV. NUMERICAL RESULTS

In this section, numerical results are graphically presented by using the previous mathematical analysis. The numerical evaluation of expression for the n-th moment requires the summation of an infinite number of terms. But, the expression converges rapidly, and thus, it can be efficiently used. The accuracy to the fourth significant digit of numerical results presented graphically in the following part of this section was accomplished by no more than twenty terms.

In Fig. 1, the normalized average signal value (R_1/y_0) is plotted as a function of shadowing severity for several values of correlation coefficient and number of BSs. As it was expected, system performance improves with a decrease of correlation coefficient and shadowing severity. The influence of correlation coefficient on the system performance is larger in the case of larger number of BSs involved and lower shadowing severity. This conclusion is made by detailed analysis of calculated numerical results. In addition, it is very interesting to observe that the average signal values at the output of systems employing macrodiversity with three correlated BSs and two uncorrelated BSs are nearly equal. Three independent BSs provide better performance in term of average signal value than four correlated BSs.

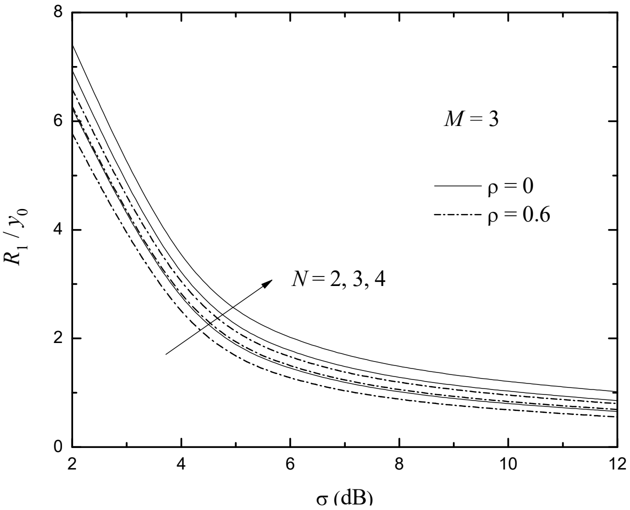

Figure 2. Normalized average signal value versus shadowing severity

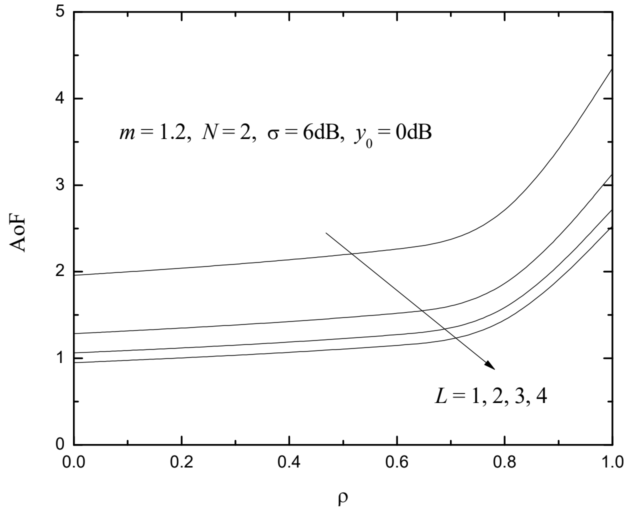

Figure 3. AoF versus correlation coefficient

TABLE I. A REALTIVE INCREASE OF AVERAGE SIGNAL VALUE FOR DIFFERENT SCENARIOS

	ρ=0		ρ=0.6	
	2→3	3→4	2→3	3→4
light shadowing	11.34 %	6.88 %	8.69 %	4.97 %
moderate shadowing	23.14 %	14.18 %	18.2 %	11.03 %
heavy shadowing	30.88 %	19 %	23.91 %	16.25 %

In Table I, a relative increase of the average signal value due to increment of BSs number from ω to υ (denoted as ω → υ) for uncorrelated and correlated scenario in environments under light (σ = 2dB), moderate (σ = 6dB) and heavy (σ = 12dB) shadowing is presented. It is evident that diversity gain is the most pronounced when signal is exposed to heavy shadowing and when BSs are independent.

Fig. 3 presents the AoF versus correlation coefficient for different number of diversity branches, i.e. antennas per BSs. It is observed that AoF decreases with an increase of diversity order, while when ρ increases, AoF also increases resulting in performance degradation. Having in mind the results presented in Table I and that the gap among the curves in Fig. 3 reduces with increase of L, we can conclude that, from the point of view of practical realization, exceedingly increasing of diversity order (number of BSs and/or antennas per BS) is not rational because an investment is not followed by the appropriate system performance improvement.

V. CONCLUSION

The closed-form expression for the n-th moment of signal at the output of the system with multibranch selection-based macrodiversity which includes multibranch MRC receivers at the micro level in composite multipath/shadowed fading environment modelled as Nakagami/gamma has been derived. Based on it, the influence of the number of diversity branches at micro and macro level, correlation coefficient and fading severity on average output signal and AoF has been considered in detail through presented numerical results.

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