Performance of wireless receiver with an AFC loop in the presence of *k-μ* multipath fading and single CCI

Caslav Stefanovic
Department of Informatics
Faculty of Natural Sciences and Mathematics, University of Pristina
Kosovska Mitrovica, Serbia.
caslav.stefanovic@gmail.com

Abstract— **In this paper the wireless communication system with the receiver consisting of an automatic frequency control (AFC) loop is considered. Proposed system is subjected to *k-μ* multipath fading environment and single interference. Performance measures such as average switching rate (ASR) and mean time to loss of lock (MTTL) are investigated. Furthermore, closed form expressions for ASR and MTTL are obtained. Numerical results are graphically presented in order to show the influence of Rician factor and fading severity parameter on ASR and MTTL.**

Keywords- automatic frequency control loop, average switching rate, co-channel interference, k-μ multipath fading, mean time to loss of lock.

I. Introduction

Wireless receiver with an automatic frequency control (AFC) loop is often integrated in wireless communication systems for the purpose to control the frequency of received signal [1]. Performances of an AFC in the presence of multipath fading and single co-channel interference (CCI) are well examined in [2-6]. It has been presented that AFC locks on the desired signal if the desired signal envelope is larger than interference envelope [2]. The degradation of the system performance of an AFC is caused when the AFC stops its tracking on the desired signal and locks on the CCI. This can happen due to the fact that amplitudes of desired signal and CCI depend on physical phenomena such as fading. Important performance measure which gives transitions from desired signal envelope to the CCI envelope and vice versa is average switching rate (ASR), already introduced in [2] and examined [2-6]. Mean time to loss of lock (MTLL) is the second performance measure also investigated [2-6], important to fully characterize system performances of an AFC. The MTTL gives the average time that an AFC stays locked on the desired signal of an AFC output.

The MTLL and ASR of an AFC in the presence of Rayleigh fading and single CCI in a closed form expressions are calculated in paper [3]. Furthermore, the MTLL and ASR of an AFC for the case of Rayleigh, Rician and Nakagami fading in the presence of a single CCI are examined in [4-5]. Case when desired signal and CCI are subjected to different fading channels are taken into account in paper [6], where the ASR and MTLL of an AFC of dissimilar fading distribution are calculated.

In this paper, ASR and MTLL of wireless communication system consisting of an AFC loop operating over *k-μ* multipath fading environment in the presence of single CCI also subjected to *k-μ* fading are evaluated. The *k-μ* distribution describes linear, line-of-sight (LOS) environments with two or more clusters. This distribution has recently been proposed [7]. Moreover, the *k-μ* distribution is general distribution, which means that for different *k-μ* fading parameters Nakagami-m, Rayleigh, and Rice fading distributions, as the special cases of *k-μ* distribution can be obtained. Therefore, in order to derive more general results *k-μ* multipath fading environment is considered. The numerical results are presented and discussed to show the influence of *k-μ* parameters on the system performances of an AFC.

II. ASR of an AFC

It has been stated that an AFC will lock on the signal with the larger amplitude between desired signal and the interferer under proposed conditions [2]. The desired signal envelope x_1 and interference envelope x_2 follow *k-μ* distribution[7]:

$$p_{x_i}(x_i) = \frac{2\mu_i (k_i+1)^{\frac{\mu_i+1}{2}}}{k_i^{\frac{\mu_i-1}{2}} e^{k_i\mu_i} \Omega_i^{\frac{\mu_i+1}{2}}} x_i^{\mu_i} \times I_{\mu_i-1}\left(2\mu_i \sqrt{\frac{k_i(k_i+1)}{\Omega_i}}\right) e^{-\frac{\mu_i(k_i+1)}{\Omega_i} x_i^2}, i=1,2 \qquad (1)$$

where k_i and μ_i are Rice factor and fading severity factor of the desired signal and interference, respectively and Ω_i is related to the local mean power of x_i. $I_v(\cdot)$ denotes the modified Bessel function of the first kind and order v [10]. The Eq. (1) can be transformed by utilization of modified Bessel function of the first kind [9, eq. (03.02.06.0002.01)] as:

$$p_{x_i}(x_i) = \frac{2\mu_i (k_i+1)^{\frac{\mu_i+1}{2}}}{k_i^{\frac{\mu_i-1}{2}} e^{k_i\mu_i} \Omega_i^{\frac{\mu_i+1}{2}}} \sum_{j_i=1}^{\infty} \left(\mu_i \sqrt{\frac{k_i(k_i+1)}{\Omega_i}}\right)^{2j_i+\mu_i-1} \times \frac{1}{j_i!\Gamma(j_i+\mu_i)} x_i^{2j_i+\mu_i-1} e^{-\frac{\mu_i(k_i+1)}{\Omega_i} x_i^2}, \qquad (2)$$

where $\Gamma(\cdot)$ is gamma function.

It has already been stated that an AFC will lock on the signal with the larger amplitude. Accordingly, the difference between two *k*-*μ* random variables is considered:

$$x = x_1 - x_2. \quad (3)$$

The average switching rate of *x* is equal to the zero level crossing rate (LCR) in positive going plus negative going directions [9]:

$$N = N_x(0) = \int_{-\infty}^{\infty} |\dot{x}| f_{x,\dot{x}}(0,\dot{x}) d\dot{x}. \quad (4)$$

The ASR of an AFC can be expressed as in [5]:

$$N = f_x(0) \int_{-\infty}^{\infty} |\dot{x}| \frac{1}{\sqrt{2\pi\Omega}} e^{\frac{-\dot{x}^2}{2\Omega}} d\dot{x} = f_x(0)\sqrt{\frac{2\Omega}{\pi}}, \quad (5)$$

where $f_z(0)$, according to [5] is equal to:

$$f_x(0) = \int_0^{\infty} p_{x_1}(x) p_{x_2}(x) dx. \quad (6)$$

Substituting Eq. (2) in Eq. (6), one has:

$$f_x(0) = \frac{\mu_1(k_1+1)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}} e^{k_1\mu_1} \Omega_1^{\frac{\mu_1+1}{2}}} \frac{2\mu_2(k_2+1)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}} e^{k_2\mu_2} \Omega_2^{\frac{\mu_2+1}{2}}}$$

$$\times \sum_{j_1=1}^{\infty} \left(\mu_1 \sqrt{\frac{k_1(k_1+1)}{\Omega_1}}\right)^{2j_1+\mu_1-1} \frac{1}{j_1!\Gamma(j_1+\mu_1)}$$

$$\times \sum_{j_2=1}^{\infty} \left(\mu_2 \sqrt{\frac{k_2(k_2+1)}{\Omega_2}}\right)^{2j_2+\mu_2-1} \frac{1}{j_2!\Gamma(j_2+\mu_2)}$$

$$\times \frac{(\Omega_1\Omega_2)^{\mu_1+\mu_2+j_1+j_2-\frac{1}{2}}}{(\mu_2(k_2+1)\Omega_1+\mu_1(k_1+1)\Omega_2)^{\mu_1+\mu_2+j_1+j_2-\frac{1}{2}}}$$

$$\times \Gamma(j_1+j_2+\mu_1+\mu_2). \quad (7)$$

Variance of $\dot{x}$ can be written as [11]:

$$\Omega = \sigma_{\dot{x}_1}^2 + \sigma_{\dot{x}_2}^2$$

$$= \pi^2 f_{m_1}^2 \frac{\Omega_1}{\mu_1(k_1+1)} + \pi^2 f_{m_2}^2 \frac{\Omega_2}{\mu_2(k_2+1)}$$

$$= \pi^2 f_m^2 \frac{(\Omega_1\mu_2(k_2+1)+\Omega_2\mu_1(k_1+1))}{\mu_1(k_1+1)\mu_2(k_2+1)}, \quad (8)$$

where f_{m_1} and f_{m_2} are maximal Doppler frequencies of the desired signal and the interference, respectively. The frequencies are assumed to be the same $f_m = f_{m_1} = f_{m_2}$.

After substituting Eq. (8) and Eq. (7) in Eq. (5), the closed form solution for ASR of an AFC in the presence of interference over *k*-*μ* multipath fading environment becomes:

$$N = \sqrt{\frac{2}{\pi}}\pi f_m \frac{\mu_1(k_1+1)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}} e^{k_1\mu_1} \Omega_1^{\frac{\mu_1+1}{2}}} \frac{2\mu_2(k_2+1)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}} e^{k_2\mu_2} \Omega_2^{\frac{\mu_2+1}{2}}}$$

$$\times \sum_{j_1=1}^{\infty} \left(\mu_1 \sqrt{\frac{k_1(k_1+1)}{\Omega_1}}\right)^{2j_1+\mu_1-1} \frac{1}{j_1!\Gamma(j_1+\mu_1)}$$

$$\times \sum_{j_2=1}^{\infty} \left(\mu_2 \sqrt{\frac{k_2(k_2+1)}{\Omega_2}}\right)^{2j_2+\mu_2-1} \frac{1}{j_2!\Gamma(j_2+\mu_2)}$$

$$\times \frac{(\Omega_1\Omega_2)^{\mu_1+\mu_2+j_1+j_2-\frac{1}{2}}}{(\mu_2(k_2+1)\Omega_1+\mu_1(k_1+1)\Omega_2)^{\mu_1+\mu_2+j_1+j_2-1}}$$

$$\times \Gamma(j_1+j_2+\mu_1+\mu_2). \quad (9)$$

III. MTTL OF AN AFC

The MTLL (*T*) of an AFC subjected to *k*-*μ* multipath fading channel in the presence of interference can be derived using the formula [5]:

$$T = \frac{2F}{N}, \quad (10)$$

where F is the probability that the x_1 is larger than x_2. According to [5], one has:

$$F = P(x_1 > x_2)$$

$$= \int_0^{\infty} dx_1 \int_0^{x_1} p_{x_1}(x_1) p_{x_2}(x_2) dx_2. \quad (11)$$

Substituting Eq. (2) in Eq. (11), the MTLL becomes:

$$F = \frac{\mu_1(k_1+1)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}} e^{k_1\mu_1} \Omega_1^{\frac{\mu_1+1}{2}}} \frac{2\mu_2(k_2+1)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}} e^{k_2\mu_2} \Omega_2^{\frac{\mu_2+1}{2}}}$$

$$\times \sum_{j_1=1}^{\infty} \left(\mu_1 \sqrt{\frac{k_1(k_1+1)}{\Omega_1}} \right)^{2j_1+\mu_1-1} \frac{1}{j_1! \Gamma(j_1+\mu_1)}$$

$$\times \sum_{j_2=1}^{\infty} \left(\mu_2 \sqrt{\frac{k_2(k_2+1)}{\Omega_2}} \right)^{2j_2+\mu_2-1} \frac{1}{j_2! \Gamma(j_2+\mu_2)}$$

$$\times \int_0^{\infty} x_1^{2j_1+2\mu_1-1} e^{-\frac{\mu_1(k_1+1)}{\Omega_1} x_1^2} \gamma\left(j_2+\mu_2, \frac{\mu_2(k_2+1)}{\Omega_2} x_1^2 \right) dx_1$$

$$\times \frac{\Omega_2}{\mu_2(k_2+1)}, \qquad (12)$$

where $\gamma(n,x)$ is the Gamma function [10]. Applying the following transformation [10]:

$$\gamma(n,x) = \frac{x^n}{n} e^{-x} F(1, 1+n; x), \qquad (13)$$

where $F(a,b;c)$ is the confluent hypergeometric function and using the integral [10]:

$$\int_0^{\infty} t^{b-1} F(a,c;kt) e^{-st} dt =$$

$$\Gamma(b) s^{-b} {}_2F_1(a,b;c;z), |s| > |k|, \qquad (14)$$

where ${}_2F_1(a,b;c;z)$ is Gauss hypergeometric function, one can obtained the closed form solution for MTLL of an AFC for the proposed model:

$$F = \frac{\mu_1 (k_1+1)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}} e^{k_1\mu_1} \Omega_1^{\frac{\mu_1+1}{2}}} \frac{\mu_2 (k_2+1)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}} e^{k_2\mu_2} \Omega_2^{\frac{\mu_2+1}{2}}}$$

$$\times \sum_{j_1=1}^{\infty} \left(\mu_1 \sqrt{\frac{k_1(k_1+1)}{\Omega_1}} \right)^{2j_1+\mu_1-1} \frac{1}{j_1! \Gamma(j_1+\mu_1)}$$

$$\sum_{j_2=1}^{\infty} \left(\mu_2 \sqrt{\frac{k_2(k_2+1)}{\Omega_2}} \right)^{2j_2+\mu_2-1} \frac{1}{j_2! \Gamma(j_2+\mu_2)} \frac{1}{j_2+\mu_2}$$

$$\times \frac{(\Omega_1 \Omega_2)^{\mu_1+\mu_2+j_1+j_2}}{(\mu_2(k_2+1)\Omega_1 + \mu_1(k_1+1)\Omega_2)^{\mu_1+\mu_2+j_1+j_2}}$$

$$\times \Gamma(j_1+j_2+\mu_1+\mu_2)$$

$$\times {}_2F_1\left(1, j_1+j_2+\mu_1+\mu_2; 1+j_2+\mu_2; \frac{\mu_2(k_2+1)}{\Omega_2} \frac{\Omega_1\Omega_2}{\mu_2(k_2+1)\Omega_1+\mu_1(k_1+1)\Omega_2}\right) \qquad (15)$$

IV. Numerical Results

Figs. 1 and 2, show the influence of different k-μ fading parameters on ASR while figs. 3 and 4, show the influence of different k-μ fading parameters on MTLL of an AFC.

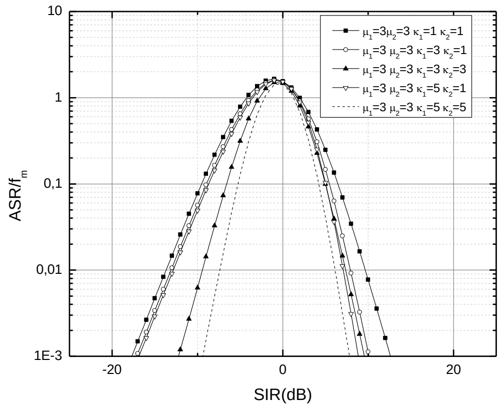

Figure 1. Normalized ASR of an AFC in k-μ multipath fading environment for different values of k_1 and k_2 and constant value of parameters μ_1 and μ_2.

In Fig 1, normalized ASR of an AFC versus the signal-to-interference ratio (SIR) for the constant fading severity parameters of the desired branch, μ_1 and CCI branch, μ_2 and different Rice factors of desired branch, k_1 and CCI branch, k_2 is presented. The ASR is normalized to f_m and the SIR is given as $SIR = 10log_{10}\frac{\Omega_1}{\Omega_2}$. By increasing the Rice factor in both branches for constant values of μ_1 and μ_2 the performance improves in the sense that the ASR decreases. In the boundary case that the Rice factors tend to infinity, the AFC locks on the signal with more power and there will be no switching.

In Fig. 2, normalized ASR of an AFC versus SIR for different parameters of μ_1 and μ_2 and constant parameter k_1 and k_2 is presented. By increasing the μ_1 and μ_2 in both branches, the performance improves in the sense that the ASR decreases.

Moreover, this figure shows that for a constant fading severity factor in the CCI branch, increasing the μ_1 causes the ASR to decrease, which contributes to the improvement in the performance for larger values of SIR, while it does not have a significant effect on the ASR for small values of SIR.

Fig. 3, shows the MTLL multiplied by f_m of an AFC for k-μ fading channel for the case when μ_1 and μ_2 are constant and k_1 and k_2 take different values. Similarly, by increasing k_1 and k_2 the MTTL increases for larger values of SIR. It can be seen that if the channel of the desired signal has larger values of k_1 (stronger LOS component), the performance of the AFC improves in the sense that the MTLL of the AFC increases for the same values of SIR.

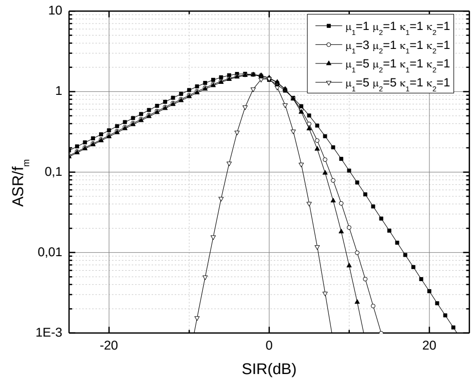

Figure 2. Normalized ASR of an AFC in *k-μ* multipath fading environment for constant values of k_1 and k_2 and different values of parameters μ_1 and μ_2.

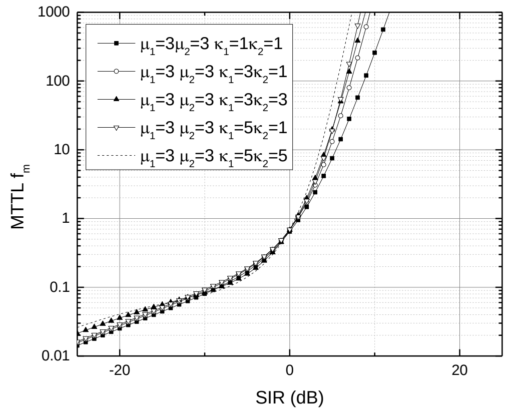

Figure 3. The MTTL of an AFC in *k-μ* multipath fading environment for different values of k_1 and k_2 and constant value of parameters μ_1 and μ_2.

In Fig 4. the MTTL multiplied by f_m versus SIR for different μ_1 and μ_2 and constant k_1 and k_2 is shown. It can be seen that if the channel of the desired signal is modeled by larger values of μ, the performance of the AFC improves so that the MTLL of the AFC increases for the same values of SIR. By comparing the MTLL curves, theoretically the best results are achieved for higher values of μ_1 and μ_2 and also for higher values of k_1 and k_2 parameters. Although, the ASR is necessary to capture the impact of the transients on the performance of a receiver, one also need to find the MTLL to fully evaluate AFC performance.

Note that for the special case when k=0, *k-μ* distribution approximates Nakagami-m distribution. Furthermore, *k-μ* distribution approximates Rice distribution for μ=1 while Rayleigh distribution is derived for k=0 and μ=1. Moreover, the obtained results are in accordance with previously published references [4-5].

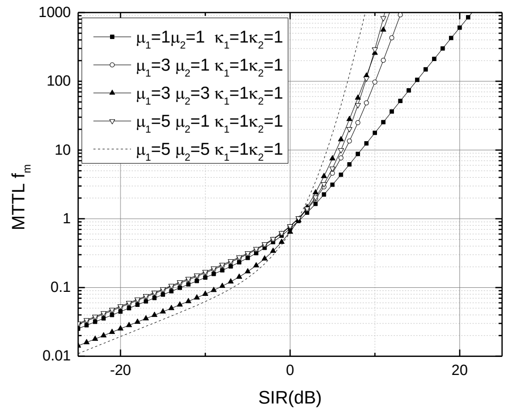

Figure 4. The MTTL of an AFC in *k-μ* multipath fading environment for constant values of k_1 and k_2 and different values of parameters μ_1 and μ_2.

V. Conclusion

The closed form expressions for the performance measure such as ASR and the MTLL of an AFC in the presence of interference over *k-μ* multipath fading are derived. Numerical results are presented and discussed to show the effect of the *k-μ* fading parameters on the performance of an AFC. For the first time, *k-μ* multipath fading is considered, to show the influence of more general fading environment on the performance of an AFC.

References

[1] H. Meyr and G. Ascheid, Synchronization in Digital Communications, Vol. I, New York: Wiley, 1990.

[2] E. J. Baghdady, Lectures on Communication System Theory, New York: McGraw-Hill, 1961.

[3] G. W. Lank and L. S. Reed, "Average Time to Loss of Lock for an Automatic Frequency Control Loop with Two Fading Signals, and a Related Probability Distribution," IEEE Trans. Inf. Theory, vol. IT-12, no. 1, pp. 73-75, Jan. 1966.

[4] A. Emad and N. C. Beaulieu, "Effect of a cochannel interferer on an automatic frequency control loop in fading channels," in Proc. IEEE Wireless Commun. Netwo. Conf. (WCNC'09), Budapest, 2009, pp 1-6.

[5] A. Emad and N. C. Beaulieu, "Performance of an AFC loop in the presence of a single interferer in a fading channel," IEEE Trans. Commun., vol. 58, no. 12, pp. 3386-3391, Oct. 2010.

[6] A. Emad and N. C. Beaulieu, "On the Performance of an Automatic Frequency Control Loop in Dissimilar Fading Channels in the Presence of Interference," IEEE Trans. Commun., vol. 59, no. 12, pp. 3234 - 3239, Dec. 2011.

[7] M.D. Yacoub, "The k- μ Distribution and the η - μ Distribution," IEEE Antennas and Propagation Magazine, Vol. 49, No . 1, pp. 68 – 81, Feb. 2007.

[8] The Wolfarm Functions Site, 2008. [Online] Available: http:/functions.wolfarm.com

[9] M. K. Simon, M. -S. Alouini, Digital Communication over Fading Channels: A Unified Approach to Performance Analysis, New York: John Wiley & Sons, Inc., 2000.

[10] I. S. Gradshteyn and I.M. Rizhik, Table of Integrals, Series, and Products, 7 ed. San Diego: Academic Press, 2007.

[11] P. Stefan, M. Stefanovic, P. Spalevic and J. Anastasov, "Fading and Interference Mitigation in Wireless Communications", CRC Press, USA, 2013.