Crossing number of macrodiversity SC receiver with three microdiversity SC receivers in the presence of Rayleigh multipath fading and Gamma shadowing

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Abstract—**Wireless communication system with macrodiversity SC reception and three microdiversity receptions is considered. Composite channel experiences Rayleigh small scale fading and correlated Gamma large scale fading. Macrodiversity SC receiver reduces Gamma long term fading effects and microdiversity SC receivers mitigate Rayleigh fading effects on system performance. Closed form expression for average level crossing rate (LCR) of macrodiversity SC receiver output signal envelope is calculated. The influence of Gamma shadowing severity parameter and Gamma shadowing correlation coefficient on average level crossing rate is analysed.**

Key Words- SC receiver; macrodiversity; Gamma shadowing; Rayleigh fading; LCR

I. INTRODUCTION

Large scale fading and small scale fading degrade system performance and limit channel capacity of wireless communication system with macrodiversity and microdiversity reception. Reflections and refractions cause multipath propagation resulting in short term fading while large obstacles between transmitter and receiver cause shadowing resulting. Short term fading can be described by using different statistical models as Rayleigh, Rician, Nakagami-*m* distributions, log-normal distribution and Gamma distributions can describe signal envelope average power in shadowing channels [1-2]. Macrodiversity technique can be used to reduce, simultaneously, multipath fading effects and shadowing effects on outage probability and bit error probability of wireless communication system.

In [3], average level crossing rate and average fade duration of macrodiversity system with macrodiversity SC receiver and two microdiversity MRC receivers operating over Gamma shadowed correlated Nakagami-*m* multipath channel are calculated. The second order statistics of macrodiversity system in the presence of Rician short term fading and correlated Gamma long term fading have been calculated [4].

In this paper, macrodiversity system with macrodiversity SC receiver and three microdiversity SC receivers is considered. Received signal experiences Gamma long term fading and Rayleigh short term fading resulting in outage probability and bit error probability degradation. Level crossing rate of microdiversity SC receiver output signal envelope is calculated. This expression is used for evaluation of average level crossing rate of macrodiversity SC receiver output signal envelope. The influence of Gamma long term fading severity and correlation coefficient of long term fading is studied.

II. LEVEL CROSSING RATE OF SC RECEIVER OUTPUT SIGNAL ENVELOPE IN THE PRESENCE OF RAYLEIGH FADING

Selection combining receiver operating over Rayleigh multipath fading environment is considered. Signal envelopes at inputs of SC receiver are denoted with x_1 and x_2 and SC receiver output signal envelope output signal envelope is denoted with x. Joint probability density function of Rayleigh random variable x_1 and its first derivative is [5]:

$$p_{x_1\dot{x}_1}\left(x_1\dot{x}_1\right)=\frac{2x_1}{\Omega_1}e^{-\frac{x_1^2}{\Omega_1}}\frac{1}{\sqrt{2\pi}\beta_1}e^{-\frac{\dot{x}_1^2}{2\beta_1^2}} \tag{1}$$

where $\Omega_1 = \overline{x_1^2}$, $\beta_1^2 = \pi^2 fm^2 \Omega_1$ and fm is maximal Dopler frequency. The average level crossing rate of Rayleigh random variable is:

$$N_{x_1} = \int_0^{\infty} dx_1 \dot{x}_1 p_{x_1\dot{x}_1}\left(x_1\dot{x}_1\right) = = \frac{2x_1}{\Omega_1} e^{-\frac{x_1^2}{\Omega_1}} \frac{1}{\sqrt{2\pi}} \beta_1 = \frac{fm\sqrt{2\pi}}{\Omega_1^{1/2}} e^{-\frac{x_1^2}{\Omega_1}} \tag{2}$$

Joint probability density function of SC receiver output signal envelope and its first derivative is:

$$p_{x\dot{x}}\left(x\dot{x}\right) = p_{x_1\dot{x}_1}\left(x\dot{x}\right) F_{x_2}\left(x\right) + p_{x_2\dot{x}_2}\left(x\dot{x}\right) F_{x_1}\left(x\right) = = 2 p_{x_1\dot{x}_1}\left(x\dot{x}\right) F_{x_2}\left(x\right) \tag{3}$$

where:

$$F_{x_2}\left(x\right) = \left(1 - e^{-\frac{x^2}{\Omega}}\right) \tag{4}$$

Average level crossing rate of SC receiver output signal envelope is:

$$N_x = \int_0^{\infty} dx \dot{x} p_{x\dot{x}}\left(x\dot{x}\right) = 2F_{x_2}\left(x\right) N_{x_1} = = \frac{2fm\sqrt{2\pi}}{\Omega_1^{1/2}} \left(1 - e^{-\frac{x^2}{\Omega}}\right) \tag{5}$$

III. LEVEL CROSSING RATE OF MACRODIVERSITY SC RECEIVER OUTPUT SIGNAL ENVELOPE

Macrodiversity system with macrodiversity selection combining diversity receiver and three microdiversity selection combining receivers is analysed. Model wireless system is shown on Fig. 1.

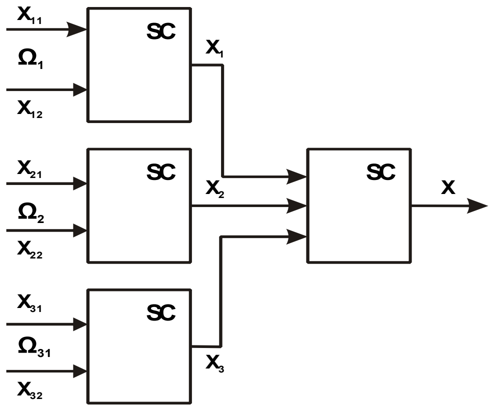

Figure 1. Model wireless system.

Received signal is subjected to correlated Gamma long term fading and Rayleigh short term fading. Signal envelopes at inputs of the first microdiversity SC receiver are denoted with x_{11} and x_{12}, at inputs of the second with x_{21} and x_{22} and that inputs of the 3[th] with x_{31} and x_{32} signal at outputs of microdiversity SC receiver are denoted with x_1, x_2 and x_3 and macrodiversity SC receiver output signal envelope is denoted with x. Signal envelope average powers at inputs of microdiversity SC receivers Ω_1, Ω_2 and Ω_3 follow correlated Gamma distribution [7].

$$p_{\Omega_1\Omega_2\Omega_3}\left(\Omega_1\Omega_2\Omega_3\right) = \frac{1}{\Gamma\left(c\right)\left(1-\rho^2\right)^2 \rho^{c-1} \Omega_0^{c+2}} \cdot \cdot \left(\Omega_1\Omega_3\right)^{\frac{c-1}{2}} e^{-\frac{\Omega_1 + \left(1-\rho^2\right)\Omega_2 + \Omega_3}{\Omega_0\left(1-\rho^2\right)}} \cdot \cdot I_{c-1}\left(\frac{2\rho}{\Omega_0\left(1-\rho^2\right)} \Omega_1^{1/2}\Omega_2^{1/2}\right) I_{c-1}\left(\frac{2\rho}{\Omega_0\left(1-\rho^2\right)}\right) = = \frac{1}{\Gamma\left(c\right)\left(1-\rho^2\right)^2 \rho^{c-1} \Omega_0^{c+2}} \sum_{i_1=0}^{\infty} \left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_1+c-1} \cdot \cdot \frac{1}{\Omega_0\left(1-\rho^2\right)} \sum_{i_2=0}^{\infty} \left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_2+c-1} \frac{1}{\Omega_0\left(1-\rho^2\right)} \cdot \cdot \Omega_1^{i_1+c-1} \Omega_2^{i_1+i_2+c-1} \Omega_3^{i_2+c-1} e^{-\frac{\Omega_1 + \left(1-\rho^2\right)\Omega_2 + \Omega_3}{\Omega_0\left(1-\rho^2\right)}} \tag{6}$$

The SC receiver selects microdiversity SC receiver with the higher signal envelope average power to provide service to user. This average level crossing rate of macrodiversity SC receiver output signal envelope is:

$$N_x = \int_0^{\infty} d\Omega_1 \int_0^{\Omega_1} d\Omega_2 \int_0^{\Omega_1} d\Omega_3 N_{x_1/\Omega_1} p_{\Omega_1\Omega_2\Omega_3}\left(\Omega_1\Omega_2\Omega_3\right) + + \int_0^{\infty} d\Omega_2 \int_0^{\Omega_2} d\Omega_1 \int_0^{\Omega_2} d\Omega_3 N_{x_2/\Omega_2} p_{\Omega_1\Omega_2\Omega_3}\left(\Omega_1\Omega_2\Omega_3\right) + + \int_0^{\infty} d\Omega_3 \int_0^{\Omega_3} d\Omega_1 \int_0^{\Omega_3} d\Omega_2 N_{x_3/\Omega_3} p_{\Omega_1\Omega_2\Omega_3}\left(\Omega_1\Omega_2\Omega_3\right) = = J_1 + J_2 + J_3 \tag{7}$$

The integral J_1 is [6]:

$$J_1 = 2fm\sqrt{2\pi}\,\frac{1}{\Gamma(c)\left(1-\rho^2\right)^2\rho^{c-1}\Omega_0^{c+2}}\cdot$$
$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_1+c-1}\frac{1}{i_1!\,\Gamma(i_1+c)}\cdot$$
$$\cdot\sum_{i_2=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_2+c-1}\frac{1}{i_2!\,\Gamma(i_2+c)}\cdot$$
$$\cdot\left(\frac{\Omega_0\left(1-\rho^2\right)}{1+\rho^2}\right)^{i_1+i_2+c}\left(\Omega_0\left(1-\rho^2\right)\right)^{i_2+c}\cdot$$
$$\cdot\frac{1}{i_1+i_2+c}\left(\frac{1+\rho^2}{\Omega_0\left(1-\rho^2\right)}\right)^{i_1+i_2+c}\sum_{j_1=0}^{\infty}\frac{1}{\left(i_1+i_2+c\right)_{(j_1)}}\cdot$$
$$\cdot\left(\frac{1+\rho^2}{\Omega_0\left(1-\rho^2\right)}\right)^{j_1}\frac{1}{i_2+c}\left(\frac{1}{\Omega_0\left(1-\rho^2\right)}\right)^{i_2+c}\cdot$$
$$\cdot\sum_{j_2=0}^{\infty}\frac{1}{\left(i_2+c+1\right)_{(j_2)}}\left(\frac{1}{\Omega_0\left(1-\rho^2\right)}\right)^{j_2}\cdot$$
$$\cdot\left(\frac{x^2\Omega_0\left(1-\rho^2\right)}{3+\rho^2}\right)^{i_1+i_2+3c/2+j_1/2+j_1/2-1/4}\cdot$$
$$\cdot K_{2i_1+2i_2+3c+j_1+j_2-1/2}\left(2\sqrt{\frac{x^2\Omega_0\left(3+\rho^2\right)}{\Omega_0\left(1-\rho^2\right)}}\right)-$$
$$\cdot\left(\frac{2x^2\Omega_0\left(1-\rho^2\right)}{3+\rho^2}\right)^{i_1+i_2+3c/2+j_1/2+j_1/2-1/4}\cdot$$
$$\cdot K_{2i_1+2i_2+3c+j_1+j_2-1/2}\left(2\sqrt{\frac{2x^2\left(3+\rho^2\right)}{\Omega_0\left(1-\rho^2\right)}}\right).\qquad(8)$$

The integral J_2 is [6]:

$$J_2 = \int_0^{\infty}d\Omega_2\int_0^{\Omega_2}d\Omega_1\int_0^{\Omega_2}d\Omega_3 N_{x_2/\Omega_2}\,p_{\Omega_1\Omega_2\Omega_3}\left(\Omega_1\Omega_2\Omega_3\right)=$$
$$=2fm\sqrt{2\pi}\,\frac{1}{\Gamma(c)\left(1-\rho^2\right)^2\rho^{c-1}\Omega_0^{c+2}}\cdot$$
$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_1+c-1}\frac{1}{\Omega_0\left(1-\rho^2\right)}\frac{1}{\Omega_0\left(1-\rho^2\right)}\cdot$$
$$\cdot\sum_{i_2=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_2+c-1}\left(\Omega_0\left(1-\rho^2\right)\right)^{i_1+i_2+2c}\cdot$$
$$\cdot\frac{1}{i_1+c}\left(\frac{1+\rho^2}{\Omega_0\left(1-\rho^2\right)}\right)^{i_1+c}\sum_{j_1=0}^{\infty}\frac{1}{\left(i_1+i_2+c\right)_{(j_1)}}\cdot$$
$$\cdot\left(\frac{1}{\Omega_0\left(1-\rho^2\right)}\right)^{j_1}\frac{1}{i_2+c}\left(\frac{1}{\Omega_0\left(1-\rho^2\right)}\right)^{i_2+c}\cdot$$
$$\cdot\sum_{j_2=0}^{\infty}\frac{1}{\left(i_2+c+1\right)_{(j_2)}}\left(\frac{1}{\Omega_0\left(1-\rho^2\right)}\right)^{j_2}\cdot$$
$$\cdot\left(\frac{x^2\Omega_0\left(1-\rho^2\right)}{3+\rho^2}\right)^{i_1+i_2+3c/2+j_1/2+j_1/2-1/4}\cdot$$
$$\cdot K_{2i_1+2i_2+3c+j_1+j_2-1/2}\left(2\sqrt{\frac{x^2\Omega_0\left(3+\rho^2\right)}{\Omega_0\left(1-\rho^2\right)}}\right)-$$
$$\cdot\left(\frac{2x^2\Omega_0\left(1-\rho^2\right)}{3+\rho^2}\right)^{i_1+i_2+3c/2+j_1/2+j_1/2-1/4}\cdot$$
$$\cdot K_{2i_1+2i_2+3c+j_1+j_2-1/2}\left(2\sqrt{\frac{2x^2\left(3+\rho^2\right)}{\Omega_0\left(1-\rho^2\right)}}\right).\qquad(9)$$

The integral J_3 is [6]:

$$J_3 = J_1\qquad(10)$$

The expression for average level crossing rate of macrodiversity SC receiver output signal envelope can be used for calculation of average fade duration of considered wireless system. Average fade duration is equal to the ratio of outage probability and average level crossing rate.

Based on previous mathematical analysis, Fig. 2 depicts the normalized average level crossing rate versus normalized signal level for several values of ρ and c for the case when macrodiversity involves the use of three base stations.

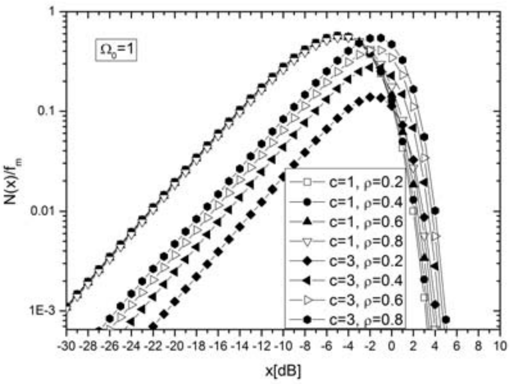

Figure 2. Normalized LCR for different parameters c and ρ.

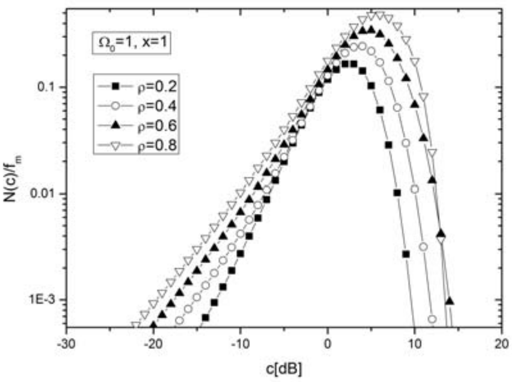

Figure 3. Normalized LCR for different parameter ρ.

At Fig. 3, average level crossing rate of macrodiversity SC receiver output signal envelope in terms of Gamma long term fading severity is presented for several values of correlation coefficient. Average level crossing rate increases as Gamma shadowing severity increases while it reaches maximum for $c=c_0$. For lower values for c, fading is more severe. The number of crossings is less but signal envelope is several times lower than threshold. For $c>c_0$ average level crossing rate decreases as Gamma long term fading severity increases.

Conclusion

Macrodiversity system with macrodiversity SC receiver and three microdiversity SC receivers operating over correlated Gamma shadowed Rayleigh multipath fading channel is considered. Received signal is subjected simultaneously to correlated Gamma long term fading and Rayleigh short term fading resulting in signal envelope power variation and signal envelope variation. Macrodiversity SC receiver Gamma large scale fading effects and microdiversity SC receivers mitigate Rayleigh small scale fading effects on system performance. In cellular mobile radio communication, microdiversity SC receiver combines signal envelopes with multiple antennas at base station and macro diversity provide signal envelopes with two or more base station geographically distributed in cell. Macrodiveristy selection combining diversity receiver selects microdiversity with the highest signal envelope average power to enables service to user. Closed form expression for average level crossing rate of microdiversity SC receiver output signal envelope is calculated. This expression is used for calculation average level crossing rate of macrodiversity SC receiver output signal envelope. This expression rapidly converges, since 10-15 terms should be summed to achieve accuracy at 4th significant digit for all values Gamma long term fading severity parameter and correlation coefficient. Numerical results are analysed to present the influence of Gamma shadowing severity parameter correlation coefficient and Rayleigh envelope average power on average level crossing rate. System performance is better for lower values of average level crossing rate. When Gamma parameter c goes to infinity shadowed multipath channel becomes Rayleigh multipath fading channel.

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