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A Model for Estimating the Solar Insolation Under Real Weather Conditions

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Abstract—**The paper presents a simple model for estimating the solar insolation on a tilted solar collector under real atmospheric condition at a particular site. The model takes the same form as simple clear day model with the parameters corrected by using long-term measurements of solar radiation in the region of Belgrade, Serbia. The optical depth and diffuse constant are determined using the method of least squares (LES) in order to model the combination of clear and cloudy conditions during the year. The validity of the proposed model has been confirmed according to one year measurements of horizontal solar radiation. Also, the applicability of the commonly used Clear Day Model has been investigated.**

Keywords-solar insolation; irradiance measurement data; least square method; optical depth; diffuse constant;

I. INTRODUCTION

Photovoltaic (PV) system consists of photovoltaic modules and other components, designed to transform the primary solar energy into electrical energy. Two most commonly encountered configurations of PV systems are: grid-connected systems (that feed power directly into the utility grid) and stand-alone systems (that supply the isolated consumers) [1].

PV systems are different from conventional electrical energy systems, because they are subjected to varying meteorological conditions. As a result, reliable insolation data are required at each site of interest to design a PV system. For a location lacking an insolation database, it is essential to have accurate models to predict the insolation. The accuracy of these models affects the design, performance, and economics of solar systems [2]. These insolation models, by very different methods, take into account the influence of various atmospheric phenomena on solar radiation. The criteria for evaluating the different insolation models should be simplicity, accuracy and the ability to use readily available meteorological data.

A wide group of insolation models are based on the assumption of clear day weather [2-4]. A set of equations are used to predict the position of the sun in the sky for a given location, as well as, the insolation on PV collector surface. However, due to the great number of cloudy days per year, the results obtained by these models do not truly represent reality, even when the models are detailed in the methods used to solve the radiative transfer problem. One of the most commonly used clear day model is the model presented by American Society of Heating, Refrigerating, and Air Conditioning Engineers (ASHRAE) which is based on empirical data collected by Threlkeld and Jordan [1,5].

Another group of insolation models uses procedures for estimating the average insolation that can be expected to strike a collector under real conditions at a particular site [6-9]. These models usually use long-term measurements of solar radiation hitting a horizontal surface. A set of equations is then used to estimate the insolation on inclined collector surfaces. Also, there are very simple cloudy sky models that need only metrological data on the total cloud amount. The performance of these models is comparable to that of the more complicated ones [10].

This paper presents a simple model for estimating the solar insolation, which parameters are determined by using the long-term measurements of horizontal solar radiation for the wider region of Belgrade, Serbia. Taking into account the clear and cloudy sky conditions, the model enables more realistic calculation of solar insolation than clear day models.

II. CLEAR DAY MODEL

Solar radiation on a tilted collector surface is the sum of components consisting of beam, sky diffuse and reflected solar radiation. For clear day, these components can be estimated by using extraterrestrial solar radiation. Expression that describes the day-to-day variation in extraterrestrial solar radiation is the following:

$$I_0 = SC \cdot \left(1 + 0.034 \cdot \cos\left(\frac{360 \cdot n}{365}\right)\right) \quad (1)$$

where SC=1367 W/m^2 is the solar constant and n is day number in the year.

Passing through the earth atmosphere the intensity of solar radiation reduces because of dispersing and absorption on atoms and ions of present gases (oxygen, hydrogen, nitrogen, ozone, carbon dioxide, etc.). Attenuation of solar radiation because it passing through the atmosphere can be present by Bouquer-Lambert low:

$$I_B = Ae^{-km} \tag{2}$$

where I_B is the beam radiation on earth surface (normal to the rays); A is the apparent extraterrestrial flux, k is the attenuation coefficient of solar radiation in the earth atmosphere, called the optical depth, m is the air mass ratio.

The apparent extraterrestrial flux, optical depth and air mass according to the ASHRAE Clear Day Model [1,5] are:

$$A = 1160 + 75\sin\left(\frac{360}{365}(n-275)\right) \quad [\mathrm{W/m^2}] \tag{3}$$

$$k = 0.174 + 0.035\sin\left(\frac{360}{365}(n-100)\right) \tag{4}$$

$$m = 1/\sin\beta \tag{5}$$

where β is the altitude angle of the sun.

The beam radiation I_{BC}, sky diffuse radiation I_{DC} and ground reflected solar radiation I_{RC} on a solar collector having tilt angle Σ from the horizontal can be calculated according to the following simplified formulas:

$$I_{BC} = I_B \cdot \cos\theta \tag{6}$$

$$I_{DC} = I_{DH} \cdot \left(\frac{1+\cos\Sigma}{2}\right) \tag{7}$$

$$I_{RC} = \rho\left(I_{BH} + I_{DH}\right)\left(\frac{1-\cos\Sigma}{2}\right) \tag{8}$$

where: θ is the incidence angle between a normal to the collector and the incoming solar beam, ρ is the coefficient of ground reflectance, I_{BH} is the beam radiation on a horizontal surface, and I_{DH} is the diffuse radiation on a horizontal surface.

The incidence angle θ between a normal to the collector and the incoming solar beam is:

$$\cos\theta = \cos\beta \cdot \cos\left(\Phi_S - \Phi_C\right) \cdot \sin\Sigma + \sin\beta \cdot \cos\Sigma \tag{9}$$

where Φ_S is the azimuth angle of the sun and Φ_C is the azimuth angle of the collector.

The beam radiation I_{BH} and the diffuse radiation I_{DH} on a horizontal surface can be calculated as follows:

$$I_{BH} = I_B \sin\beta \tag{10}$$

$$I_{DH} = C \cdot I_B \tag{11}$$

where C is the diffuse constant.

A convenient approximation for diffuse constant C according to the ASHRAE Clear Day Model [1,5] is:

$$C = 0.095 + 0.04 \cdot \sin\left(\frac{360}{365} \cdot \left(n-100\right)\right) \tag{12}$$

The total solar radiation on a tilted solar collector is:

$$I_C = I_{BC} + I_{DC} + I_{RC} \tag{13}$$

III. MODEL FOR CLEAR AND CLOUDY SKY CONDITIONS

Due to the great number of cloudy days per year, models for estimating the average insolation on the solar collector under real conditions at a particular site are needed. These models can be developed by using long-term measurement data of solar radiation hitting a horizontal surface. In order to use these data for tilted surfaces, the total solar radiation I_H on a horizontal surface should be divided into beam radiation I_{BH} and diffuse radiation I_{DH}:

$$I_H = I_{BH} + I_{DH} \tag{14}$$

Procedures for decomposing total horizontal radiation on its components are based on clearness index K_T which is defined as:

$$K_T = \frac{\overline{I}_H}{\overline{I}_0} \tag{15}$$

where $\overline{I}_H$ is the average horizontal insolation at the site and $\overline{I}_0$ is the average horizontal extraterrestrial insolation.

The average horizontal extraterrestrial insolation can be calculated as follows:

$$\overline{I}_0 = \left(\frac{24}{\pi}\right) SC\left[1 + 0.034\cos\left(\frac{360n}{365}\right)\right]\left(\cos L \cos\delta \sin H_{sr} + H_{sr} \sin L \sin\delta\right) \tag{16}$$

where H_{sr} is the sunrise hour angle, L is the latitude of the site and δ is the solar declination.

The solar declination and sunrise hour angle are defined as:

$$\delta = 23.4\sin\left(\frac{360}{365}\left(n-81\right)\right) \tag{17}$$

$$H_{SR} = \arccos\left(-\mathrm{tg}L \cdot \mathrm{tg}\delta\right) \tag{18}$$

A number of authors proposed the expressions for diffuse-to-total radiation ratio as a function of clearness index. According to Liu-Jordan, diffuse-to-total horizontal radiation ratio is:

$$\frac{I_{DH}}{I_H} = 1.39 - 4.027K_T + 5.531K_T^2 - 3.108K_T^3 \tag{19}$$

The clearness index in Liu-Jordan formula is used for decomposing the total horizontal insolation (the solar energy per horizontal surface) in its components. The Liu-Jordan formula can be also used for decomposing the total horizontal irradiance (the solar radiation power per horizontal surface) on its components, as shown in (19).

The beam and diffuse components of horizontal irradiance can be determined as long-term variables $I_{BH\,i}$ and $I_{DH\,i}$, if the long-term measurements of horizontal irradiance $I_{H\,i}$ is used (i=1,...,N where N is the total number of measurement values). Such variables can be used for determination of optical depth and diffuse constant by applying the method of least squares (LES).

According to (2), (5) and (10), the optical depth can be expressed as follows:

$$k = \frac{1}{m}\ln\left(\frac{A}{I_B}\right) = \frac{1}{m}\ln\left(\frac{A}{I_{BH}\sin\beta}\right) = \frac{1}{m}\ln\left(\frac{A\cdot m}{I_{BH}}\right) \tag{20}$$

According to the set of one year measurement data for the horizontal irradiance $I_{H\,i}$, (i=1,...,N), the set of one year values for beam horizontal irradiance $I_{BH\,i}$ (i=1,...,N) can be obtained applying the Liu-Jourdan formula. For example, for ten-minute measurment interval over one year measuring period, the total number of measurents will be N=52560. The set of one year values for beam horizontal irradiance $I_{BH\,i}$ (i=1,...,N) can be used for determining the set of one year values for optical depth k_i:

$$k_i = \frac{1}{m_i}\ln\left(\frac{A_i\cdot m_i}{I_{BH\,i}}\right) \tag{21}$$

where $I_{BH\,i}$ is the i-th value of beam horizontal irradiance obtained according to the i-th value of measured total horizontal irradiance $I_{H\,i}$, m_i is the i-th value of air mass, and A_i is the i-th value of apparent extraterrestrial flux calculated in the same intervals over one year measuring period.

The same model as (4) for optical depth can be assumed:

$$k = k_1 + k_2\sin\left(\frac{360}{365}(n-100)\right) \tag{22}$$

where k_1 and k_2 are constants that should be determined for real weather conditions during the year.

The constants k_1 and k_2 can be determined by applying LES method. The following LES minimization problem can be defined:

$$\sum_{i=1}^{N}\left(\left(k_1 + k_2\sin f_i\right) - k_i\right)^2 \to \min \tag{23}$$

where:

$$\sin f_i = \sin\left(\frac{360}{365}\left(i\frac{365}{N} - 100\right)\right) \tag{24}$$

The optimum estimate of coefficients k_1 and k_2 which defined the optical depth are:

$$k_1 = \frac{\sum_{i=1}^{N}k_i\cdot\sum_{i=1}^{N}(\sin f_i)^2 - \sum_{i=1}^{N}k_i\cdot\sin f_i\cdot\sum_{i=1}^{N}\sin f_i}{n\cdot\sum_{i=1}^{N}(\sin f_i)^2 - \left(\sum_{i=1}^{N}\sin f_i\right)^2} \tag{25}$$

$$k_2 = \frac{n\cdot\sum_{i=1}^{N}k_i\cdot\sin f_i - \sum_{i=1}^{N}k_i\cdot\sum_{i=1}^{N}\sin f_i}{n\cdot\sum_{i=1}^{N}(\sin f_i)^2 - \left(\sum_{i=1}^{N}\sin f_i\right)^2} \tag{26}$$

The same methodology can be used for estimating the diffuse constant. The same model as (12) for diffuse constant can be assumed:

$$C = C_1 + C_2\cdot\sin\left(\frac{360}{365}\cdot(n-100)\right) \tag{27}$$

The constants C_1 and C_2 can be determined by applying LES method. The following LES minimization problem can be defined:

$$\sum_{i=1}^{N}\left(\left(C_1 + C_2\sin f_i\right)I_{B\,i} - I_{DH\,i}\right)^2 \to \min \tag{28}$$

where $I_{DH\,i}$ is the i-th value of diffuse horizontal irradiance obtained according to the i-th value of measured total horizontal irradiance $I_{H\,i}$, and $I_{B\,i}$ is the i-th value of beam irradiance obtained by substituting estimated optical depth in (2) or according to the i-th value of beam horizontal irradiance $I_{BH\,i}$:

$$I_{B\,i} = I_{BH\,i}\cdot m_i \tag{29}$$

The optimum estimate of coefficients C_1 and C_2 which defined the diffuse constant are:

$$C_1 = \frac{\sum_{i=1}^{N}I_{DH\,i}\,I_{B\,i}\cdot\sum_{i=1}^{N}I_{B\,i}^2(\sin f_i)^2 - \sum_{i=1}^{N}I_{DH\,i}\,I_{B\,i}\cdot\sin f_i\cdot\sum_{i=1}^{N}I_{B\,i}^2\sin f_i}{\sum_{i=1}^{N}I_{B\,i}^2\cdot\sum_{i=1}^{N}I_{B\,i}^2(\sin f_i)^2 - \left(\sum_{i=1}^{N}I_{B\,i}^2\sin f_i\right)^2} \tag{30}$$

$$C_2 = \frac{\sum_{i=1}^{N}I_{DH\,i}\,I_{B\,i}\sin f\cdot\sum_{i=1}^{N}I_{B\,i}^2 - \sum_{i=1}^{N}I_{DH\,i}\,I_{B\,i}\cdot\sum_{i=1}^{N}I_{B\,i}^2\sin f_i}{\sum_{i=1}^{N}I_{B\,i}^2\cdot\sum_{i=1}^{N}I_{B\,i}^2(\sin f_i)^2 - \left(\sum_{i=1}^{N}I_{B\,i}^2\sin f_i\right)^2} \tag{31}$$

The optical depth and diffuse constant are determined according to (22) and (27), respectively. By using the measured data and method of least squares for modeling these two quantities, all sky condition during the year are taken into account. Knowing optical depth and diffuse constant, the solar irradiance striking a collector can be calculated. By integrating the solar irradiance striking a collector on a daily basis, the average daily insolation on a tilted collector is obtained.

IV. EXPERIMENTAL RESULTS

The analysis is based on measurements of horizontal solar irradiance in the region of Belgrade, Serbia. The measurements with ten-minute resolution have been carried out over 2009, by using NRG data acquisition system and Li-Cor pyranometer. The total horizontal irradiance has been decomposed into its beam and diffuse components according to the calculated clearness index and Liu-Jourdan formula. The measured values of horizontal irradiance and its beam and diffuse components over the year are presented in Fig. 1.

The optimum values of coefficients k_1 and k_2 which define the optical depth, according to (25) and (26), are k_1=0.600 and k_2=-0.130. The optimum estimate of optical depth is:

$$k = 0.600 - 0.130 \cdot \sin\left(\frac{360}{365}(n-100)\right) \tag{32}$$

The optimum values of coefficients C_1 and C_2 which define the diffuse constant, according to (30) and (31), are C_1=0.370 and C_2=-0.069. The optimum estimate of diffuse constant is:

$$C = 0.370 - 0.069 \cdot \sin\left(\frac{360}{365} \cdot (n-100)\right) \tag{33}$$

The estimated values of optical depth and diffuse constant over the year are presented in Fig. 2. The values of optical depth and diffuse constant calculated according to ASHRAE Clear Day Model are also presented in the Fig. 2. Obviously, the attenuation and diffusion of solar radiation is more intensive in the case of real sky conditions (which include clear and cloudy sky) compared to the case of only clear sky conditions over the year. Also, the attenuation and diffusion of solar radiation, according proposed model, is more intensive during the winter as a consequence of more cloudy weather during the winter.

In order to test the proposed model independently of the measured data used for developing the model, another set of one-year measuring data (for year 2014) has been used for testing the model. The values of average daily beam, diffuse and total insolation on a tilted collector during the year, according to measurement data, according to proposed model and according to ASHRAE Clear Day Model, are presented in Fig. 3, in Fig. 4 and in Fig. 5, respectively. The collector optimum tilt angle of 34^0 for Belgrade has been assumed, according to [12].

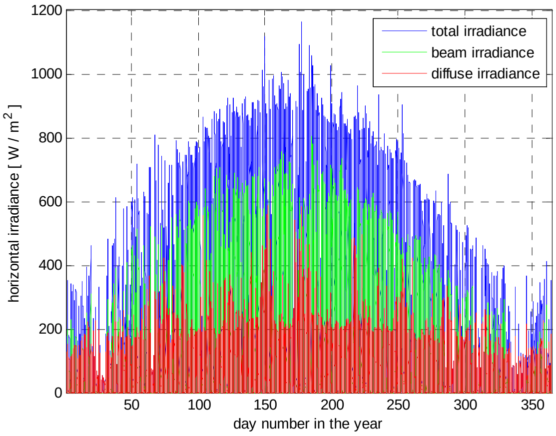

Figure 1. The measured horizontal irradiance and its beam and diffuse components

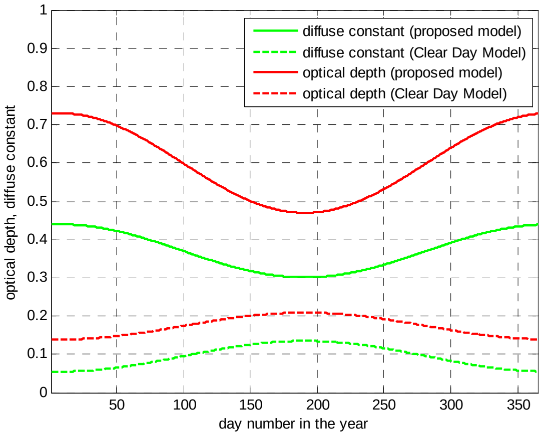

Figure 2. The values of optical depth and diffuse constant during the year calculated according to the proposed model and according to the ASHRAE Clear Day Model

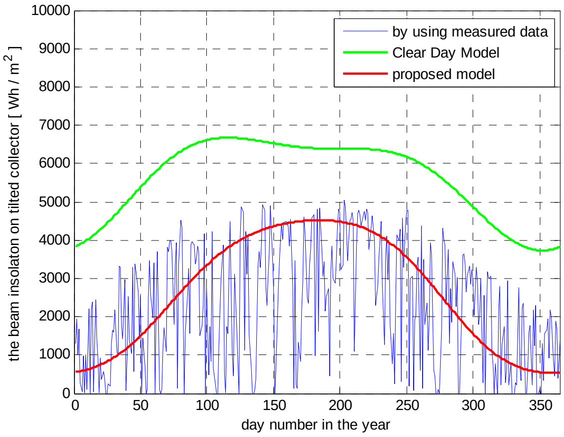

Figure 3. The average daily beam insolation on a tilted collector during the year calculated according to the measurement data, according to the proposed model and according to the ASHRAE Clear Day Model

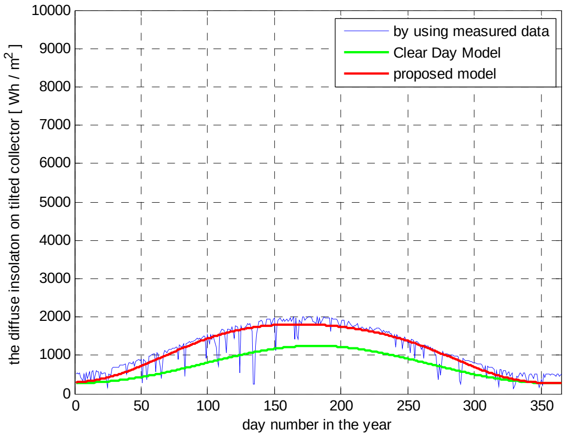

Figure 4. The average daily diffuse insolation on a tilted collector during the year calculated according to the measurement data, according to the proposed model and according to the ASHRAE Clear Day Model

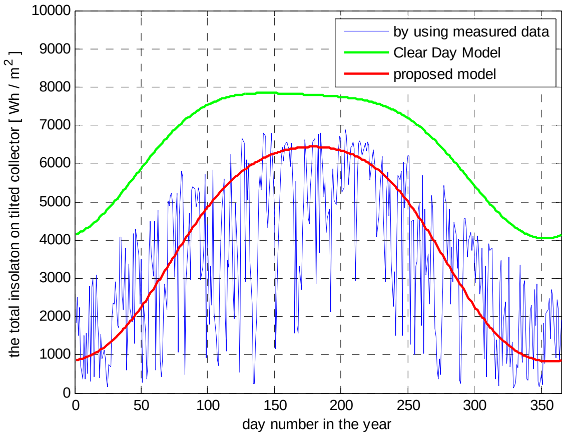

Figure 5. The average daily total insolation on a tilted collector during the year calculated according to the measurement data, according to the proposed model and according to the ASHRAE Clear Day Model

According to ASHRAE Clear Day Model, the greater values for average daily beam insolation and the smaller values for average daily diffuse insolation are obtained. These errors are somewhat compensated in average daily total insolation, but this method is still much less accurate then the proposed one. Furthermore, according to Fig. 5, it is clear that the Clear Day Model does not fit the insolation even for clear days. As can be seen, the estimated total insolation according to Clear Day Model is much bigger than the clear day peak values of total insolation obtained according to measured data.

The values of annual average daily beam, diffuse and total insolation on a tilted collector, according to measurement data, according to proposed model and according to ASHRAE Clear Day Model, are presented in Table. 1. The relative error in prediction the annual average daily total insolation according to Clear Day Model reaches the unacceptable value of 85%. According to proposed model, the relative error in the prediction of annual average daily total insolation is 11%, which is caused by annual changes in sky conditions. The better prediction of average daily total insolation can be achieved if two or more year measurement data are used for developing the model. This requires perennial measurements of solar irradiance.

TABLE I. ANNUAL AVERAGE DAILY BEAM, DIFFUSE AND TOTAL INSOLATION ON A TILTED COLLECTOR

Annual average daily insolation	**Model**		
	according to measurements	***Clear Day Model***	***proposed model***
daily beam insolation [Wh/m^2]	2271	5607	2679
daily diffuse insolation [Wh/m^2]	1155	742	1116
daily reflected insolation [Wh/m^2]	53	88	60
daily total insolation [Wh/m^2]	3479	6436	3854
relative error for total insolation [%]	0	85.01	10.78

V. CONCLUSION

The paper presents a simple model for estimating the solar insolation on a tilted solar collector. The model is based on the long-term measurements of horizontal solar irradiance carried out in the region of Belgrade, Serbia. By applying the LES method, the optical depth and diffuse constant are determined. Taking into account the clear and cloudy sky conditions, the proposed model enables more realistic calculation of solar insolation than the Clear Day Model. The relative error in the prediction of annual average daily total insolation reaches the value of 85% when the Clear Day Model is used, and the value of 11% when the proposed model is used.

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