Mathematical model of electromagnetic vibratory exciter with incremental motion

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Abstract—**Electromagnetic vibratory drives offer easy and simple control of the mass flow of particulate and bulk materials. In comparison to mechanical drives (pneumatics, hydraulics, or inertial), these have a more simple construction and they are compact, robust, and reliable in operation. The absence of wearing mechanical parts, such as gears, cams belts, bearings, eccentrics, or rotational actuators, makes the vibratory drives (conveyors, bins, hoppers, feeders etc.) as most economical equipment. Vibrations of bins or hoppers, in which the particulate material is placed, produced by electromagnetic linear exciter induce the movement of material particles, so that they resemble a highly viscous liquid, and the material becomes easier to transport and to dose. A mathematical model of electromagnetic vibratory exciter is presented in this paper. Based on this mathematical model is generate the simulation circuit for analysis of vibratory exciter behavior in the stationary and transient regimes. Simulation and experimental results and their comparisons are exposed in this paper. Experimental results are recorded on practically realized vibratory drive for efficient discharge of collecting hoppers in electrostatic precipitator plants.**

Keywords-Electromagnetic vibratory exciter, electromagnetic force, inductor, modelling, simulations, collecting hoppers, electrostatic precipitators.

I. INTRODUCTION

Electromagnetic vibratory exciter are found application in vibration systems of conveying bulk and particulate materials, wherein the optimal transport achieved in a wide range frequency of load carrier elements (vibrating troughs, conveyors, spiral conveyors, etc..).The oscillations of the load carrier element are produced by excitation force from a vibrating electromechanical actuator i.e. electromagnetic vibratory exciter (EVEX). In the case of extraction bulk and particulate materials from the bins or hoppers is usually applied as exciters EVEX actuators.

The motivation for this research is a result of practical experience in systems for evacuation of fly ash in thermal power plants. In addition to the high voltage power supply of electrostatic precipitators (ESP) and motion drive of the electrode rappers, the system for ash removal from the collecting hoppers is very important. There are two commonly accepted systems for ash removal: hydraulic transport done by water and pneumatic transport done by gas, usually air [1]. For both cases, it can be said that the flow of ash material can be further enhanced by the use of EVEX placed at the bottom of the hopper, as shown in Fig.1.

Although collecting hoppers are common in use, the internal flow of the material is not well understood, relying heavily on empirical information on maintaining the operation. For example, when a particulate ash is agglutinate at the exit of the hopper, it may cause arching and prevent flow. To remedy the situation, vibration may be used, sometimes in the crude form of a hammer, to perturb the material and initiate the flow [2]. Alternatively, the hopper may be equipped with an EVEX unit to continuously jiggle of the hopper wall. These activators must be carefully designed to enhance the flow and prevent further settling, arching or clogging of the material.

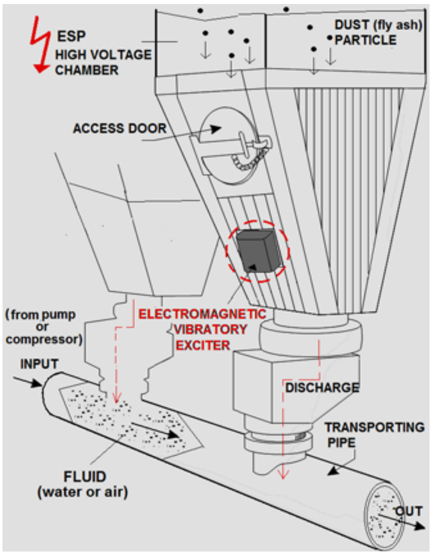

Figure 1. Dispositions of electromagnetic vibratory exciter in ash collecting system of ESP hopper.

Electromagnetic drives having EVEX offer easy and simple control for the mass flow conveying materials. In comparison to all previously mentioned drives, these have a more simple construction and they are compact, robust, and reliable in operation. The absence of wearing mechanical parts, such as gears, cams belts, bearings, eccentrics, or motors, makes vibratory hopper, conveyors or feeders most economical equipment [3-5]. Application of EVEX applied to discharge hoppers in combination with power converters provides a significant flexibility in fulfilling the requirements for efficient flow and conveyance of dust or ash from the collecting hoppers of ESP's.

II. THE CONSTRUCTION OF ELECTROMAGNETIC VIBRATORY EXCITERS

All main types of vibratory exciter can be seen as two-mass systems. The majority of them generate harmonic excitation forces, while some types generate transmitting impact pulses. The EVEX can be single- or double-stroke construction. In the single-stroke type, there is an electromagnet, whose armature is attracted in one direction, while the reverse stroke is completed by restoring elastic forces. In the two-stroke type, two electro-magnets, which alternately attract the armature in different directions, are used [6]. In Fig.2 two of the most common single-stroke constructions are shown. One of them has armature on its active side, while inductor is on its reactive side, as shown in Fig. 2(a). The other construction is set vice versa, as shown in Fig. 2(b).

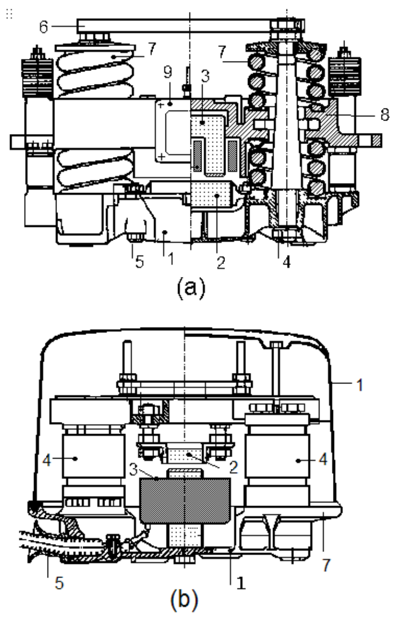

Figure 2. The construction of the EVEX.

(a)-Inductor on reactive side. (b) Inductor on active side.

Fig. 2(a) shows the structure of EVEX used for power above 1 kW and which generates higher excitation force. An inductor (3) of electromagnet, composed of iron dynamo plates, is located on the free side of exciter. This part is attached to the active part (1), over the spring system (7) and of the bracket (8). Anchor i.e. armature (2) made of soft iron is attached to active side of exciter. Transverse joints (6) connect systems of springs, while nut (4) adjusts the pressurising of the springs. The active part is connected to the load across the screw bolt. Power supplying of inductor is achieved through connecting box (9).

Fig.2 (b) shows the structure which is used for dozers and vibratory hoppers of small power (200W below). In this arrangement, armature (2) is attached to the free side of the exciter, over the nut (6). Inductor (3) is attached to the active part (7) of the exciter. Power of inductor is achieved through connecting cable (5). The part that carries the armature is attached via elastic elements i.e. spacers (4) on both sides to active part of exciter. Metal case (1) protects the EVEX construction from moisture and fine particles.

Below it will be discussed in more detail mathematical model and will be presented to the dynamic equations of motion for a vibratory exciter from Fig.1 (a).

III. PRESENTATION OF DYNAMIC MODEL OF ELECTROMAGNETIC VIBRATORY EXCITER

The mathematical model of EVA is based on presentation in Figs. 2(a) and 3 with details. An electromagnet is connected to an ac source and the reactive section is mounted on an elastic system of springs. During each half period when the maximum value of the current is reached, the armature is attracted, and at a small current value it is repelled as a result of the restoring elastic forces in springs. Therefore, vibratory frequency is double frequency of the power supply. These reactive vibratory exciters can also operate on interrupted pulsating (dc) current. Their frequency in this case depends on the pulse frequency of the dc. The mechanical force, which is a consequence of this current (created by electromechanical conversion in the exciter), is transmitted through the springs to the mass load.

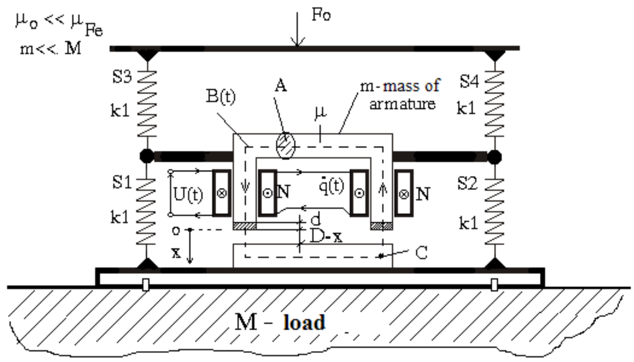

Figure 3. Cross section of EVEX for modelling and analysis

It is assumed that the mass of load M is much greater than the mass m of a movable reactive section. Let us suppose that springs are identically constructed, with stiffness k_1 and prestressed with action adjustable force F_0. This force is used for setting air gap value in the actuator. The nonlinearity of spring elements is neglected. Total equivalent damping coefficient of system springs is β . The movement of the inductor is restricted in the x -direction. At $t = 0$ (initial moment) gravitational force is compensated by prestressing spring forces.

It is supposed that the ferromagnetic material has a very high permeability μ_{Fe} (the reluctance of the magnetic core path can be ignored) compared to μ_0 of the air gap and bronze disk. Consequently, all the energy of the magnetic field is stored in air gap and bronze. The area of cross section of the air gap is A. The air gap length in the state of static equilibrium is D. The bronze disk with thickness d does not permit inductor to form a complete magnetic circuit of iron; in other words, it inhibits "gluing" of inductor, which is undesirable. Fringing and leakage at the air gap can be neglected too. In order to reduce eddy currents loss, magnetic core is laminated. Also, the magnetic circuit operation in the linear region of $B(H)$ magnetization curve, with adequate limitation of the current value, is assumed. Excitation coils are connected to the voltage source $u(t)$. The source has its own resistance R_s, while the excitation coils (hereafter termed "coil") have their own resistance R_c. The current in the $2N$-turns excitation coil is noted as $i(t)$.

Ampere's law for the reference direction of path C, as shown in Fig.3, will be applied according to following equation:

$$H_x \cdot (D-x) + H_b \cdot d = N \cdot i \tag{1}$$

Where are H_x – magnetic intensity in air gap and H_b - magnetic intensity in bronze.

The flux density is:

$$B = \mu_0 \cdot H_x = \mu_0 \cdot H_b \tag{2}$$

Substituting this expression into equation (1), the flux density is:

$$B = \frac{\mu_o \cdot N \cdot i}{D+d-x} \tag{3}$$

The flux in bronze and air gap is:

$$\Phi = B \cdot A = \frac{\mu_o \cdot N \cdot A \cdot i}{D+d-x} \tag{4}$$

The total flux is:

$$\lambda(\dot{q}, x) = 2 \cdot N \cdot \Phi = \frac{2 \cdot \mu_o \cdot N^2 \cdot A \cdot i}{D+d-x} \tag{5}$$

The state function of the magnetic co energy is:

$$W_m^* = \int_0^i \lambda(i,x)di \tag{6}$$

The solution of this integral is:

$$W_m^*(i,x) = \frac{\mu_o \cdot N^2 \cdot A \cdot i^2}{(D+d-x)} \tag{7}$$

The total system co energy is:

$$\frac{1}{2}m\dot{x}^2 + W_m^*(i,x) \tag{8}$$

Equation (7) can be usefully shown as:

$$W_m^*(i,x) = a \cdot \frac{i^2}{D+d-x} \tag{9}$$

where constant a is:

$$a = \mu_0 \cdot N^2 \cdot A \tag{10}$$

Excitation coils are connected to the voltage source $u(t)$. The source has its own resistance R_S, while the excitation coils (hereafter termed "coil") have their own resistance R_C. The movement of the inductor is limited by the springs force. The external impacts are: voltage $u(t)$, and gravitational force mg. The expression for the total energy of the system is [7-8]:

$$\frac{1}{2}m\dot{x}^2 + W_m^*(i,x). \tag{11}$$

Based on a static diagram of springs forces in the absence of the electromagnetic force (Fig.3) will be determined function of co energy systems and the conditions of static equilibrium for any position of inductor:

$$\sum_i F_i = mg + F_1 - F_2 = 0 \tag{12}$$

Forces in springs are given as:

$$F_1 = k \cdot x_a \tag{13}$$

$$F_2 = k \cdot x_b. \tag{14}$$

The static equilibrium equation is given as:

$$mg + k \cdot x_b - k \cdot x_a = 0 \tag{15}$$

or:

$$mg = k \cdot (x_a - x_b). \tag{16}$$

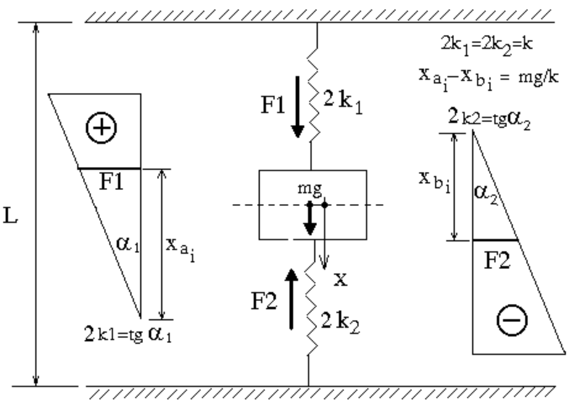

Figure 4. Static diagram forces in EVEX

The potential energy of the springs system is :

$$A_p = \frac{1}{2}k \cdot (x+x_a)^2 + \frac{1}{2}k \cdot (x-x_b)^2 \tag{17}$$

where the state function of electric energy is zero, because there is no electrostatic energy is accumulated in the system. Based on the previously derived relations, we can write the Lagrangian of the whole system [7-8]:

$$L(\dot{x}, x, i) = \frac{1}{2} m\dot{x}^2 + \frac{ai^2}{D+d-x} - A_p \, . \tag{18}$$

Rayleigh's dissipate function is given by following relation [7-8]:

$$R(\dot{x}, i) = \frac{1}{2}\beta \cdot \dot{x}^2 + \frac{1}{2}(R_s + R_c)i^2 \, . \tag{19}$$

The equation of motion for the mechanical subsystem using Lagrange equation can be written as:

$$\frac{d}{dt}\left[\frac{\partial L(\dot{x}, x, i)}{\partial \dot{x}}\right] - \frac{\partial L(\dot{x}, x, i)}{\partial x} + \frac{\partial R(\dot{x}, i)}{\partial \dot{x}} = F_x \, . \tag{20}$$

Considering the equations (15), (18), (19) and (20), the equations of motion for a mechanical subsystem becomes

$$m\ddot{x} + \beta \cdot \dot{x} + 2kx = \frac{a \cdot i^2}{(D+d-x)^2} \tag{21}$$

The equation of motion for electromagnetic subsystem is:

$$\frac{d}{dt}\left[\frac{\partial L(\dot{x}, x, i)}{\partial i}\right] + \frac{\partial R(\dot{x}, i)}{\partial i} = F_q \tag{22}$$

or

$$\frac{2a}{D+d-x}\dot{i} + (R_s + R_c)i + \frac{2a \cdot i \cdot \dot{x}}{(D+d-x)^2} = u(t) \tag{23}$$

The first term of the equation (23) is voltage that has been induced from current change in the circuit of the coil. Inductance of the circuit is the function of the inductor's position. The second term presents voltage drop on the equivalent resistance. The third term is actually induced electromotive force, which is a consequence of exertion of the mechanical sub-system on the electromagnetic subsystem.

Differential equations of motion for EVEX with incremental motion, can definitely write in the form:

$$m\ddot{x} + \beta \cdot \dot{x} + 2kx = \frac{a \cdot i^2}{(D+d-x)^2} \tag{24}$$

$$\frac{2a}{D+d-x} \cdot \frac{di}{dt} + (R_s + R_c)i + \frac{2a \cdot i \cdot \dot{x}}{(D+d-x)^2} = u(t) \tag{25}$$

These two equations are the basis for the development of an EVEX simulation model.

IV. THE SIMULATION MODEL OF EVEX

Simulation circuit of the EVEX is created on the basis of previously derived differential equations. Functional diagram is shown in Fig. 5, upon which the simulation model is based. Mechanical quantities are shown with equivalent electric quantities according table of electromechanical analogy for inverse system [9-10]. A simulation model is generated in program package PSPICE.

On the basis of this model is analyzed operation of the vibratory exciter. In given model has integrated all the nonlinearity describing in presented dynamic equations. Based on this model is created sub-circuit that is inserted into the existing library components in PSPICE. Note that the inductance $L(x)$ of the EVEX coil is variable function of the coordinate x, i.e. air gap $z = D + d - x$.

Electromagnetic force F_{em} is a function of the coordinate x and actual value of current i. For a small displacement, which are present in reality (x << D + d) this force is practically only dependent on the square of the current i. A counter electro motive force (EMF) keeps the balance of the voltage and it is a function of the speed of the EVEX movable part. The resistance of the windings to be taken into account regardless of its value is much smaller than the minimum value of a variable inductive resistance $\omega L(x)$. The internal resistance of voltage sources is neglected.

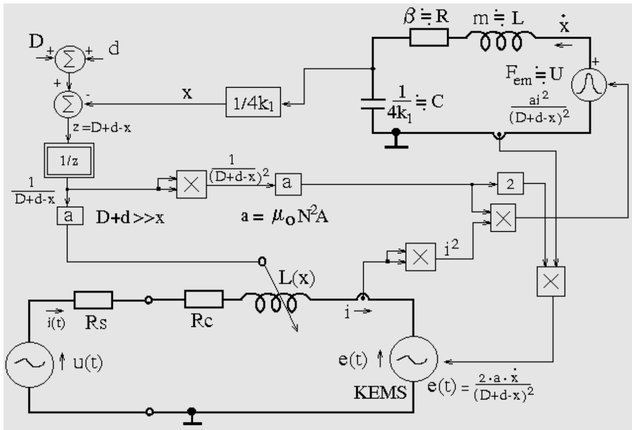

Figure 5. PSPICE simulation scheme of EVEX

V. THE CMPARISON OF SIMULATION AND EXPERIMENTAL RESULTS

This section presents the simulation results on a generated simulation model of the EVEX. In addition, as a confirmation of the simulation results, are presented the measured values on real vibratory hopper exciter as used in practical applications on ESP stations on thermal power plants. It is observed the characteristic values: current of vibratory exciter and acceleration of reactive part (inductor) of EVEX. Measured values in the simulation model are recorded in a simulation program PSPICE PROBE, while the measurements on real models made through the appropriate current probe and acceleration sensor:

- measuring of current is performed by current probe amplifier AM503-TEKTRONIX, current range 0-50A, 10MHz bandwidth, direct output to oscilloscope

-measuring of acceleration is performed by inductive acceleration sensors HBM-B12/1000 with voltage transmitter; acceleration range 0-20g, frequency 0-200Hz; voltage transmitter output signal 0-10Vdc.

In the measurements was used a digital storage oscilloscope PM3350 /50MHz/100Ms/sec- PHILIPS. In the order to compare the behavior of the simulation model and the real EVEX in transient regimes, was performed an experiment in which realized regime: turning on (activation), operation in a given time interval and turning off. Technical data of the EVEX which is tested are given in Tables I and II.

TABLE I- ELECTRICAL DATA AND PARAMETERS OF EVEX

rated voltage	$U_n = 220V, 50H$
rated current	$I_n = 4A$
coil resistance	$R_c = 5.6\Omega$
coil inductance (“quiet inductance”)	$L_c = 160mH$

TABLE II- MECHANICAL PARAMETERS OF EVEX

mass of movable part	$m = 3.2kg$
total stiffness	$k = 4k_1 = 1kN / mm$
width of air gap, adjustable in range	4-10mm
bronze thickness	$d = 1mm$

In this chapter are compared obtained simulation and experimental results. The voltage on the vibration exciter is set to the RMS value of 120V, and frequency of 50Hz. It is observing a time interval of 1s. In the real EVEX, the supplying voltage is achieved by autotransformer 380/0-440V, 50Hz. The voltage on the coil of EVEX is adjusted to 120V RMS. During a given time interval of 1s is achieved: activation i.e. turn-on, continuous operation and turn-off of EVEX.

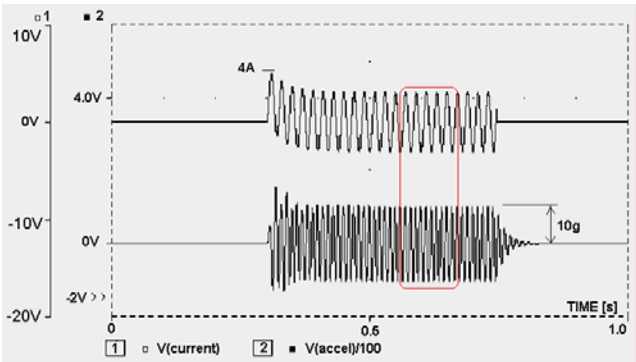

Figure 6. PSPICE simulations of transient states and continuous operation of EVEX; current gain Ki=1A/V , acceleration gain Ka=10g/V.

Fig. 6 shows the PSPICE simulation of transient states and continuous operation obtained on the simulation circuit, presented on Fig. 5. During time interval $\Delta t = 1s$ are shown waveforms of electromagnet coil current and acceleration of the movable part of the EVEX. Fig. 7 shows the simulation waveforms of current and acceleration of EVEX, related to the continuous operation (indicated in Fig. 6 with a rectangular outline).

In Figs.8 and 9 are obtained the same waveforms as in the previous case, but the real EVEX whose real characteristics and parameters are given in Tables I and II.

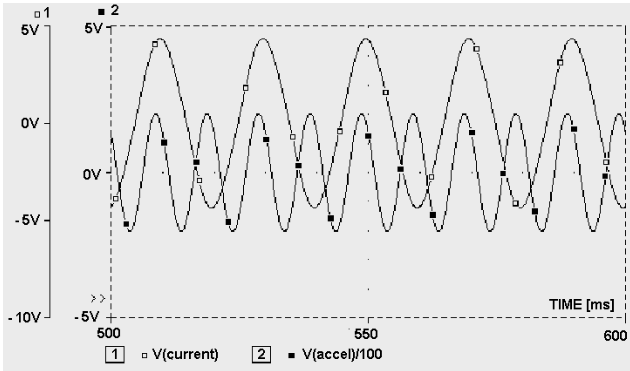

Figure 7. The simulations of continuous operation of EVEX (zoomed rectangular area in Fig.6) in program package PSPICE; current gain Ki=1A/V, acceleration gain Ka=10g/V.

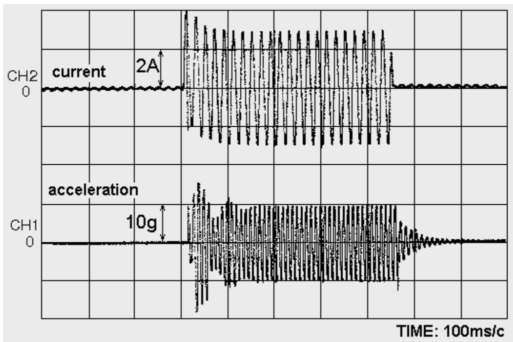

Figure 8. Oscilloscopic waveforms current and moving part acceleration, of the real EVEX in transient regimes; current gain 2A/c, acceleration gain 10g/c

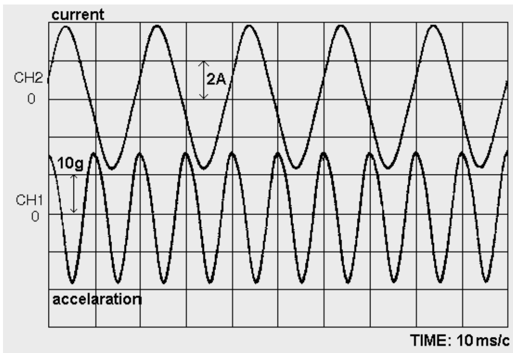

Figure 9. Oscilloscopic waveforms current and moving part acceleration, of the real EVEX in continuous operation regime; current gain 2A/c, acceleration gain 10g/c

Based obtained results, it is observed a very good agreement between the simulation and experimental results. In the following will be commented in more detail, the obtained results.

VI. DISCUSION ON OBTAINED SIMULATION AND EXPERIMENTAL RESULTS

At the moment of turning-on the vibrating exciter is observed that the maximum current is $I_{VE\max} = 4A$ which is due to the fact that in this moment induced counter electromotive force is equal to zero. In addition, there is a maximum in the value of the acceleration ($a = 15g$) of the movable part of the exciter. This peak is actually a consequence of the impact force which is proportional to the square of the actual value of the current exciter.

When moving part begins to oscillate due to the effect of the impact force, in the solenoid coil induce a voltage, or a counter electromotive force (CEMF), which is proportional to the speed of oscillation. These voltages keep the balance to power supply network, so that in a steady state, RMS current of EVEX is equal $I_{VE} = 2A$. The frequency of current is equal to mains frequency i.e. 50Hz. Under these conditions, the acceleration amplitude of the EVEX movable part is about $10g$ ($g = 9.81m/s^2$). The frequency of oscillation of the moving part is 100Hz, since the frequency of the excitation mechanical force is 100Hz. This is a due to the fact, that this force is a square function of the EVEX current.

During EVEX turning-off the, it's current almost immediately drops to zero value, but his moving part continues to oscillate with a time constant of damping approximately $T_d \approx 0.12s$. These oscillations are due to the accumulated potential energy in the elastically elements (springs) and moving part of the EVEX. Observed attenuation is a result of losses in the elastic elements of the vibration exciter, but also due to the resistance to movement of movable parts. Based on the foregoing, it is possible to determine the equivalent damping coefficient based on the time of damping and mass of moving part of the actuator. For a given mass of the EVEX moving parts of 3kg, and damping time of $T_d \approx 0.12s$, the result is that the coefficient of damping is $\beta = 2m/T_d = 50N/m/s$.

VII. CONCLUSION

In this paper is deriving a mathematical model of the EVEX based on the presented construction of conventional vibratory exciter having electromagnetic drive. This model is described by two dynamic differential equations: first, that describes the mechanical movement and another that describes the electrical behavior of the electromagnet of EVEX.

Based on the mathematical model is a generated PSPICE simulation circuit, were carried out simulations of the EVEX in regimes: turning-on, stationary operation and at the end turning-off. It is monitored the inductor (electromagnet) current and acceleration of the movable part of the vibratory exciter. The simulation results were compared with experimentally obtained on a real vibratory exciter having electromagnetic drive, which is used in practical applications (such as the vibratory hoppers on ESP plants). Based on the measured values of the real EVEX, it is found that the generated simulation model really describes the state of the real exciter and as such can be applied to other simulation PSPICE schemes.

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