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# Features of Power Transfer Distribution Coefficients in power System Networks

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*Abstract*—**PTDF (Power Transfer Distribution Factor) has recently become a common notion from the power systems network analysis glosary. It is often used for available capacity assessment in large power system networks. PTDF is also used in power system operations and planning, in planning of power/energy transactions, in planning the network additions, in steady-state security applications and concepts, as well as for economy/security assessment and enhancement (ATC, flowgates). Some aspects of PTDF applications are presented. PTDFs are relatively easy to calculate, similarly to well-known distribution factors. Calculations of PTDFs are based on the DC power flow method. Therefore, the PTDF based algorithms comprise the homogenity and additivity properties of linear networks enhancing the applicability and speed of the algorithms for power flow monitoring and control used by TSOs in power systems.**

***Key words-power systems; steady-state security; PTDF; DC power flow; power markets; transactions/transfers of power/energy***

## I. INTRODUCTION

The power systems security in deregulation (restructuring) could rightfully be approached if the common steady state security concept is retained [1], [2]. Paralel to that, many new concepts are introduced in practice subtly offsetting the tradittional point of view on steady state security. For example, the grid (power system network) is separated from the generation, the open access to the grid is allowed, transactions are 'running' through the grid causing sometimes network congestions. Congestion management is now an important topic, as well as the ATC (Available Transfer Capacity), PTDF (Power Transfer Distribution Factors), etc [3], [4]. Most importantly, in the deregulated power systems environment, the grid is subjected to the new stressors, the multiple bilateral transactions/ transfers of power as a result of trade. Trades are the market based contracts made among the suppliers and consumers of power. In theory, a bilateral transaction is modeled as a pair of power injections of identical intensity and opposite sign, one positioned in the node of supplier and the other in the consumer's node. It is well known that the power system is steady-state secure if it is robust and not exposed to risk. The conclusion is based on the cyclic examining of the system response to the pre-defined list of contingencies (Contingency Analysis, CA). The power system is defined through the AC or the DC power flow solution. If a single limit violation on any of the system's components (line/transformer) occures, the system state is singled-out as non-secure. In total, transactions of power do not affect the system balance of power expressed in the Tellegen's theorem [5]. In the DC power flow, transactions do not affect the total balance of power, they are 'in and out' through the 'national' grid, not even contributing to network losses. However, they certainly affect the power flow patterns in the grid changing them relative to the 'base-case state', i.e. the power flow pattern of injections and flows in the network prior to transactions. The base-case and other states should also pass the CA check to be declared secure. Then, the presence of bilateral transactions may jeopardize the system security. Their influence on the rest of the system can be determined with superposition by using the PTDF. Generally speaking, the following conditions may occur in the system, and which are modeled in a similar way: outages of some of the components in a system in which there are transactions, decrease of the amount of the transaction that has arisen as a control action issued by the TSO to preserve the security of the system, increase the volume of transactions as a result of the accepted offer on the spot managed market, multilateral transaction etc. PTDF's can be easily calculated, and with them it is possible to get quick and approximate condition of the system in the presence of a new transaction which might even increase the value of Available Transfer Capacity (ATC) [6] and [7], and thus raise security to a higher level. These factors provide ability to control power flow through the system, but this is outside the scope of this paper. The aim of this paper is to present a part of the phenomenology related to this concept, and so comes to its essence.

## II. PTDF

PTDF is a factor which represents the percentage of change in power flow through network branch (*i-j*) due to the existence of a new transaction in the system (from bus X to bus Y). This can be shown analytically as:

$$PTDF_{i-j,X\to Y} = \frac{\begin{pmatrix}\text{Change in power flow through network branch i - j}\\ \text{due to the transaction from X to Y}\end{pmatrix}}{\text{Power of transaction from x to Y}} \quad (1)$$

Let us by *A* (adjustments) denote the matrix of the distribution factors or sensitivities [8], [9]. It provides the

Ukoliko postoje sponzori, navesti ih ovdje. U protivnom, obrisati ovaj text box. (sponsors) incremental change of active power flow on transmission lines caused by an incremental change of the injection at some node in the network. These factors depend on the choice of balancing nodes, topology and network parameters but they are independent of the injection and power flow through system. Distribution factor for the line $\varepsilon$, on which flow of active power is changed by $\Delta P_{\varepsilon}$ due to the change of injection (production / consumption) at node B, for $\Delta P_B$, is:

$$A_{\varepsilon,B} = \Delta P_{\varepsilon} / \Delta P_B, \qquad \Delta P_B > 0 \tag{2}$$

Based on the (2) another expression of the activee power flow through line $\varepsilon$, in an arbitrary base regime could be written, in the form of

$$P_{\varepsilon}^{0} = \sum_{B \neq R} \left\{ A_{\varepsilon,B} \cdot P_B \right\} \tag{3}$$

where R represents the balancing node, where

$$P_R = -\sum_{B \neq R} P_B \qquad \wedge \quad A_{\varepsilon,R} = 0 \tag{4}$$

Injections into the network (production) are positive, while injections from the network (consumption) are likewise negative, while $P_B$>0. Suppose now that a new transaction is supermiposed to the base case, e.g. between the nodes X and Y, $PT_{X \to Y}$>0. Achieving this transaction will change the power flow on line $\varepsilon$ for $\Delta PT_{\varepsilon, X \to Y}$, which can be written as

$$\begin{aligned} P_{\varepsilon} &= P_{\varepsilon}^{0} + \Delta P_{\varepsilon,X \to Y} = \\ &\sum_{B \neq R,X,Y} \left\{ A_{\varepsilon,B} \cdot P_B \right\} + A_{\varepsilon,X}(P_X + PT_{X \to Y}) + A_{\varepsilon,Y}(P_Y - PT_{X \to Y}) = \\ &\sum_{B \neq R} \left\{ A_{\varepsilon,B} \cdot P_B \right\} + (A_{\varepsilon,X} - A_{\varepsilon,Y}) PT_{X \to Y} \end{aligned} \tag{5}$$

From (5) follows the definition of the relative share of power transactions $PT_{X \to Y}$ in power flow of line $\varepsilon$

$$PTDF_{\varepsilon,X \to Y} = A_{\varepsilon,X} - A_{\varepsilon,Y} \tag{6}$$

or

$$PTDF_{\varepsilon,X \to Y} = \Delta P_{\varepsilon,X \to Y} / PT_{X \to Y} \tag{7}$$

From (6), apparently PDTF can be calculated knowing the network topology and system parameters. It turns out that they are independent of the choice of the balancing node. If these factors are calculated using the AC load flow, then the advantage of the DC approach that (6) is a function of network parameters only is lost.

Transmission route, so called *flowgates*, are part of the network in the usual routes of transmission, with prominent values of PTDF. In the TSO practice, that transmission route will be most affected by the transactions and their identification is of great importance for the security of the system.

Errors of a few percent in the PTDF can be fatal in the final outcome, as shown by the system blackout post-mortem analysis in Northeast Ohio, USA, on August 14, 2003 at 16:05 [10].

## III. Calculation and Simulation

PTDF could be calculated at least on two different ways. The simplest algorithm is: (1) using a power flow method (AC or DC) power flows are calculated; (2) incremental power injections are placed in the end-nodes of the transaction; (3) a new power flow for the new case is obtained; (4) divide the change of the power flow through network branches caused by the observed injection with the amount of that injection [1]. This gives PTDF for observed element for the considered transaction. This procedure is correct by definition, and accurate values are obtained in the case when AC power flow is used. By using DC power flow and system parameters, PTDF are calculated in a faster way [11]. Since modern references usually do not go that deeply as to start with the generalized branch and graph theory (see, for the opposite example [12], [13]), an alternative 'off-the-cuff' approach is used, and the validity of the approach is here tried. So we have

$$PTDF = B^{f} \cdot B_r^{-1} \tag{8}$$

where $B^f$ is reduced matrix of branch network susceptances. This is a reduced branches/nodes incidence matrix, rows multiplied by the connecting line susceptance. $B_r$ is a reduced matrix of nodes' susceptances, as in the DC power flow, where reduction implies that the first row and column of the network susceptance matrix is deleted, if the balancing (slack) node is no.1. This procedure is repeated on several networks and topologies to get to the convention for internal use. Information about used networks can be found in [12], [14].

Fig. 1 a. shows a base case for the observed system in the absence of the transaction where for the slack node is the node 1. Fig 1 b. shows the distribution of power flows through lines when transaction between nodes 2 and 3 is present. In order to check the independency of PTDFs with the change of the slack node and with other transactions, Fig. 1 c. shows the same network where node 2 is slack, and transaction is between nodes 1 and 3. Equation (9) and (10) show how to calculate PTDF for all lines of the observed system for considered transaction by using (7). On the other hand, equations (11) and (12) show the same procedure by using expression (8). As was expected, the same results were obtained. Equation (9) and (11) are valid for the observed system when the slack was no. 1 node, and equations (10) and (12) when the slack was node no. 2.

From theese examples it can be seen that the PTDF neither depends on the choice of the slack node nor on the amonut of the transaction. Fig. 2.a. shows the modified system with five nodes [12] in whitch there was

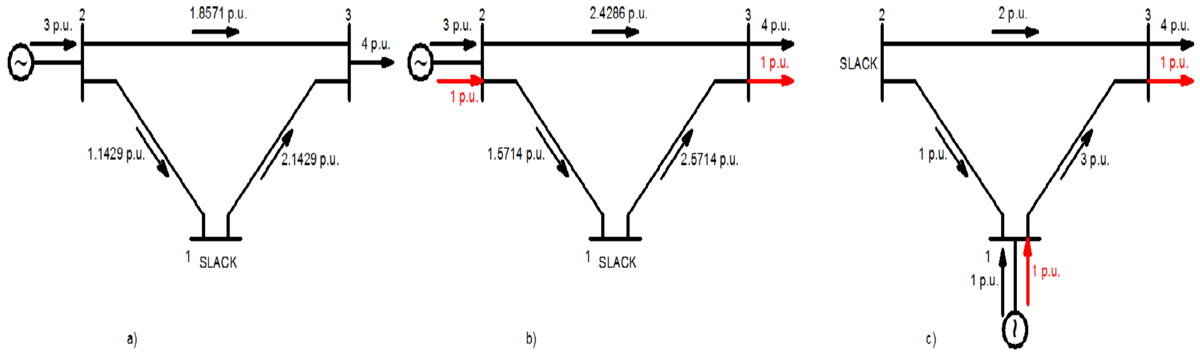


Figure 1. a. Base network, slack node is 1; b. Trans. 1 p.u., slack node is 1; c. Trans. 1 p.u., slack node is 2.

outage of one line, i.e. line 2-4. In addition to the base case, which can be seen in [4], there is a transaction of 200 MW between node 2 and node 4. As a result of line 2-4 outage and the presence of transaction between nodes 2 and 4, there is a thermal overload of line 3-4, thermal limits being 150 MW. To maintain the security of the system it is necessary to reduce the amount of the transaction. The amount by which the transaction is to be reduced to make the system secure can be determined by uzing the PTDF. Responding PTDF's for considered transaction are given in (13).

$$\begin{aligned} PTDF(1-2,2\to 3) &= \frac{P_{1-2}-P_{1-2}^{base}}{P_{2\to 3}} = -0.4285 \\ PTDF(2-3,2\to 3) &= \frac{P_{2-3}-P_{2-3}^{base}}{P_{2\to 3}} = 0.5715 \\ PTDF(1-3,2\to 3) &= \frac{P_{1-3}-P_{1-3}^{base}}{P_{2\to 3}} = 0.4285 \end{aligned} \tag{9}$$

$$\begin{aligned} PTDF(1-2,1\to 3) &= \frac{P_{1-2}-P_{1-2}^{base}}{P_{1\to 3}} = 0.1429 \\ PTDF(2-3,1\to 3) &= \frac{P_{2-3}-P_{2-3}^{base}}{P_{1\to 3}} = 0.1429 \\ PTDF(1-3,1\to 3) &= \frac{P_{1-3}-P_{1-3}^{base}}{P_{1\to 3}} = 0.8571 \end{aligned} \tag{10}$$

$$\begin{aligned} PTDF = B^f B_r^{-1} &= \begin{bmatrix} 2 & 0 \\ -2 & 2 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 2 & -8 \end{bmatrix}^{-1} \\ &= \begin{Vmatrix} 0 & -0.5714 & -0.1429 \\ 0 & 0.4286 & -0.1429 \\ 0 & -0.4286 & -0.8571 \end{Vmatrix} \end{aligned} \tag{11.1}$$

$$PTDF(2\to 3) = \begin{Vmatrix} -0.4285 \\ 0.5715 \\ 0.4285 \end{Vmatrix} \quad PTDF(1\to 3) = \begin{Vmatrix} 0.1429 \\ 0.1429 \\ 0.8571 \end{Vmatrix} \tag{11.2}$$

$$\begin{aligned} PTDF = B^f B_r^{-1} &= \begin{bmatrix} -2 & 0 \\ 0 & 2 \\ -6 & 6 \end{bmatrix} \begin{bmatrix} -8 & 6 \\ 6 & -8 \end{bmatrix}^{-1} \\ &= \begin{Vmatrix} 0.5714 & 0 & 0.4286 \\ -0.4286 & 0 & -0.5714 \\ 0.4286 & 0 & -0.4286 \end{Vmatrix} \end{aligned} \tag{12}$$

$$PTDF(2\to 3) = \begin{Vmatrix} -0.4285 \\ 0.5715 \\ 0.4285 \end{Vmatrix} \quad PTDF(1\to 3) = \begin{Vmatrix} 0.1429 \\ 0.1429 \\ 0.8571 \end{Vmatrix}$$

$$PTDF = \begin{array}{c} {\scriptstyle 1-2} \\ {\scriptstyle 1-3} \\ {\scriptstyle 2-3} \\ {\scriptstyle 2-5} \\ {\scriptstyle 3-4} \\ {\scriptstyle 4-5} \end{array} \begin{Vmatrix} 0 & -0.8582 & -0.5672 & -0.5896 & -0.7687 \\ 0 & -0.1418 & -0.4328 & -0.4104 & -0.2313 \\ 0 & 0.0970 & -0.3881 & -0.3507 & -0.0522 \\ 0 & 0.0448 & -0.1791 & -0.2388 & -0.7164 \\ 0 & -0.0448 & 0.1791 & -0.7612 & -0.2836 \\ 0 & -0.0448 & 0.1791 & 0.2388 & -0.2836 \end{Vmatrix} \tag{13}$$

$$PTDF(2\to 4) = \begin{array}{c} {\scriptstyle 1-2} \\ {\scriptstyle 1-3} \\ {\scriptstyle 2-3} \\ {\scriptstyle 2-5} \\ {\scriptstyle 3-4} \\ {\scriptstyle 4-5} \end{array} \begin{Vmatrix} -0.2687 \\ 0.2687 \\ 0.4478 \\ 0.2836 \\ 0.7164 \\ -0.2836 \end{Vmatrix}$$

Based on the (2) another expression

TSO shall issue an order to reduce the amount of transaction, calculate how much transaction will be curtailed and check the new power flow pattern. TSO left on purpose the line 3-4 on the very thermal limit (so, there is a congestion in the network), counting that in the observed hour injection in system will not change. PTDF for line 3-4 is 0.7164, so that the necessary reduction in transaction between nodes 2 and 4 in order to achieve the desired reduction of 31.79 MW on line 3-4

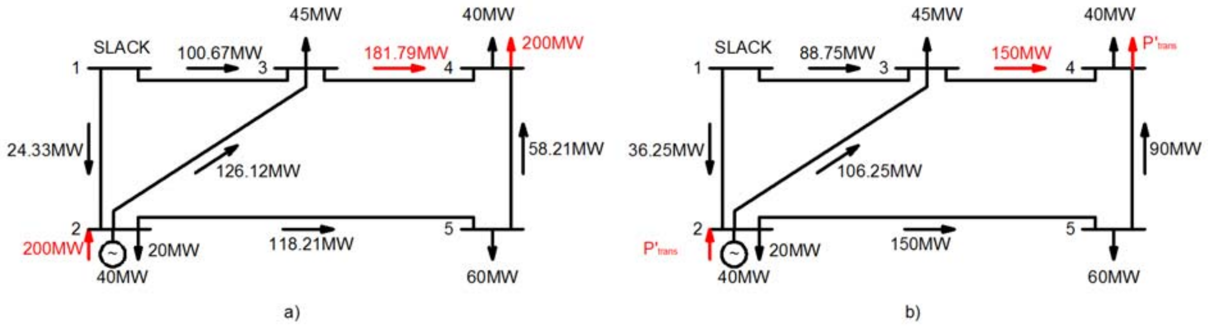


Figure 2. a. Power flow through system [7], .b. Snapshot after relief.

can be calculated as:

$$\Delta P_{2-4}^{inj} = \frac{\Delta P_{3-4}}{PTDF(2 \to 4{,}3-4)} = \frac{31.79}{0.7164} = 44.37\ \mathrm{MW} \tag{14}$$

The calculated value is delivered to the market operator which uses the managed spot market to balance supply and demand. This control does not require the involvement of the local generators. It is assumed that the transaction came from the interconnection to the national system.

## IV. TEST SYSTEM

The entire procedure will be tested on the standard IEEE 24 RTS bus system [14]. Firstly, it is necessary to calculate power flows through system lines. Next step is formation of the matrix from (8). On Fig. 3 *B* matrix is shown. Fig. 4 gives the value of matrix $B^f$. By using these values it is possible to calculate PTDF for any transaction. Table 1 shows the PTDFs for all lines, for transaction between nodes 21 and 8.

## V. CONCLUSION

This paper shows a method based on the use of PTDF for the assessment and enhancement of the available transfer capacity for the transmission of power/energy. The invariance of the PTDF of choice of the balancing node and transactions is shown. The method is suitable for the future transactions planning, in the long term network planning, wherever the concept of security of the power system matters, as well as with market issues containing the economic and security aspects (ATC, *flowgates*). Examples of calculation of PTDF are also shown in this paper. Presented PTDF are coefficients calculated on the DC power flow platform. This approach speeds up the algorithm for monitoring and power flow control in network operated by the TSO. ‘Open access’ to networks, as

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -88.51 | 71.942 | 4.7348 | 0 | 11.834 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 71.942 | -85.04 | 0 | 7.8927 | 0 | 5.2083 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 4.7348 | 0 | -25.06 | 0 | 0 | 0 | 0 | 0 | 8.4034 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.919 |
| 4 | 0 | 7.8927 | 0 | -17.54 | 0 | 0 | 0 | 0 | 9.6432 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 11.834 | 0 | 0 | 0 | -23.16 | 0 | 0 | 0 | 0 | 11.325 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 5.2083 | 0 | 0 | 0 | -21.74 | 0 | 0 | 0 | 16.529 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | -16.29 | 16.287 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 16.287 | -28.4 | 6.0569 | 6.0569 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 8.4034 | 9.6432 | 0 | 0 | 0 | 6.0569 | -47.94 | 0 | 11.919 | 11.919 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 11.325 | 16.529 | 0 | 6.0569 | 0 | -57.75 | 11.919 | 11.919 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.919 | 11.919 | -68.77 | 0 | 21.008 | 23.923 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.919 | 11.919 | 0 | -45.75 | 10.352 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.561 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 21.008 | 10.352 | -42.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.561 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 23.923 | 0 | 0 | -49.63 | 0 | 25.707 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -117.9 | 57.803 | 0 | 0 | 0 | 0 | 40.816 | 0 | 0 | 19.268 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 25.707 | 57.803 | -165.4 | 38.61 | 0 | 43.29 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 38.61 | -117.6 | 69.444 | 0 | 0 | 0 | 9.4967 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 69.444 | -146.4 | 0 | 0 | 76.923 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 43.29 | 0 | 0 | -93.8 | 50.505 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 50.505 | -143.1 | 0 | 0 | 92.593 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 40.816 | 0 | 0 | 76.923 | 0 | 0 | -132.5 | 14.749 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9.4967 | 0 | 0 | 0 | 14.749 | -24.25 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.561 | 11.561 | 0 | 0 | 0 | 0 | 0 | 0 | 92.593 | 0 | 0 | -115.7 | 0 |
| 24 | 0 | 0 | 11.919 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 19.268 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -31.19 |

Figure 3. Elements od matrix *B*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 -2 | -71.9 | 71.94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 -3 | -4.73 | 0 | 4.735 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 -5 | -11.8 | 0 | 0 | 0 | 11.83 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 -4 | 0 | -7.89 | 0 | 7.893 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 -6 | 0 | -5.21 | 0 | 0 | 0 | 5.208 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 -9 | 0 | 0 | -8.4 | 0 | 0 | 0 | 0 | 0 | 8.403 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 -24 | 0 | 0 | -11.9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.92 |
| 4 -9 | 0 | 0 | 0 | -9.64 | 0 | 0 | 0 | 0 | 9.643 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 -10 | 0 | 0 | 0 | 0 | -11.3 | 0 | 0 | 0 | 0 | 11.33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 -10 | 0 | 0 | 0 | 0 | 0 | -16.5 | 0 | 0 | 0 | 16.53 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 -8 | 0 | 0 | 0 | 0 | 0 | 0 | -16.3 | 16.29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 -9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -6.06 | 6.057 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 -10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -6.06 | 0 | 6.057 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 -11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -11.9 | 0 | 11.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 -12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -11.9 | 0 | 0 | 11.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 -11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -11.9 | 11.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 -12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -11.9 | 0 | 11.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 -13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -21 | 0 | 21.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 -14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -23.9 | 0 | 0 | 23.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 -13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -10.4 | 10.35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 -23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -11.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.56 | 0 |
| 13 -23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -11.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11.56 | 0 |
| 14 -16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -25.7 | 0 | 25.71 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 -16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -57.8 | 57.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 -21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -40.8 | 0 | 0 | 0 | 0 | 0 | 40.82 | 0 | 0 | 0 |
| 15 -24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -19.3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 19.27 |
| 16 -17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -38.6 | 38.61 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 -19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -43.3 | 0 | 0 | 43.29 | 0 | 0 | 0 | 0 | 0 |
| 17 -18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -69.4 | 69.44 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 -22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -9.5 | 0 | 0 | 0 | 0 | 9.497 | 0 | 0 |
| 18 -21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -76.9 | 0 | 0 | 76.92 | 0 | 0 | 0 |
| 19 -20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -50.5 | 50.51 | 0 | 0 | 0 | 0 |
| 20 -23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -92.6 | 0 | 0 | 92.59 | 0 |
| 21 -22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -14.7 | 14.75 | 0 | 0 |

Figure 4. Elements od matrix $B^f$

TABELA I. PTDF FOR TRANSACTION FROM 21 TO 8

| Line | PTDF | Line | PTDF | Line | PTDF | Line | PTDF |
|---|---|---|---|---|---|---|---|
| 1-2 | 0.0388 | 6-10 | 0.0265 | 11-14 | -0.3979 | 16-19 | 0.3501 |
| 1-3 | -0.0806 | 7-8 | 0 | 12-13 | -0.0830 | 17-18 | -0.3538 |
| 1-5 | 0.0418 | 8-9 | -0.5117 | 12-23 | -0.2214 | 17-22 | -0.0560 |
| 2-4 | 0.0123 | 8-10 | -0.4883 | 13-23 | -0.1287 | 18-21 | -0.3538 |
| 2-6 | 0.0265 | 9-11 | -0.1988 | 14-16 | -0.3979 | 19-20 | 0.3501 |
| 3-9 | 0.1714 | 9-12 | -0.1292 | 15-16 | 0.3381 | 20-23 | 0.3501 |
| 3-24 | -0.2521 | 10-11 | -0.2448 | 15-21 | -0.5902 | 21-22 | 0.0560 |
| 4-9 | 0.0123 | 10-12 | -0.1752 | 15-24 | 0.2521 | | |
| 5-10 | 0.0418 | 11-13 | -0.0457 | 16-17 | -0.4098 | | |

a prerequisite principle to electricity markets operations introduces power flows often as a consequence of market activities. These extreme situations result from bilateral transactions stemming from the differences of electricity prices in the regions. PTDF matrix calculations are used to control (manage) the congestion of transmission capacities, to allocate resources, and to allocate the security responsibilities.

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