

# Modeling of Humanoid Systems Using Deductive Approach

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**Abstract**—It is a well known fact that the growth of technology has radically changed our approach to biosciences and medicine. What is interesting is that in the last decade we have witnessed a reverse influence – a trend towards “biologically inspired” solutions to technical problems. This leads to a true symbiosis between bio and technical sciences. A good example is the intersection and overlapping of three distinct fields: sports, medicine, and robotics. This paper intends to apply sophisticated methods developed for mathematical modeling of humanoid robots to real human motions. A general simulation system is realized: following a deductive principle the algorithm considers particular human/humanoid motions (like those occurring in different sports) as being just special cases of a general motion and impact theory.

**Keywords:** modeling; humanoid systems; deductive approach; link; contact.

## I. INTRODUCTION

At the very beginning of our discussion, we pose a question that is crucial for the topic: does one need a general model of dynamics of human and humanoid robot motion, and why? Although the positive answer might seem obvious, we think that some supporting comments are useful. This general approach is useful for several reasons. From a purely academic point of view, general methods are always seen as a final target. From a commercial point of view, a software package that can cover a diversity of motions is a more economic solution than several specialized packages.

However, a final argument still comes from the field of humanoid robotics. Humanoids, being the future of robotic science, are becoming more and more human-like in all aspects of their functioning. It is expected that they will replace humans in variety of tasks. Thus, it is generally accepted that their shape and motion should be based on biomechanical principles [1, 2]. Due to the complexity and high requirements imposed on such robots, their control system has to utilize the dynamic model. So, the control, the design, and the simulation, strongly require general dynamic models that will make humanoid robots capable of handling the increasing diversity of expected tasks.

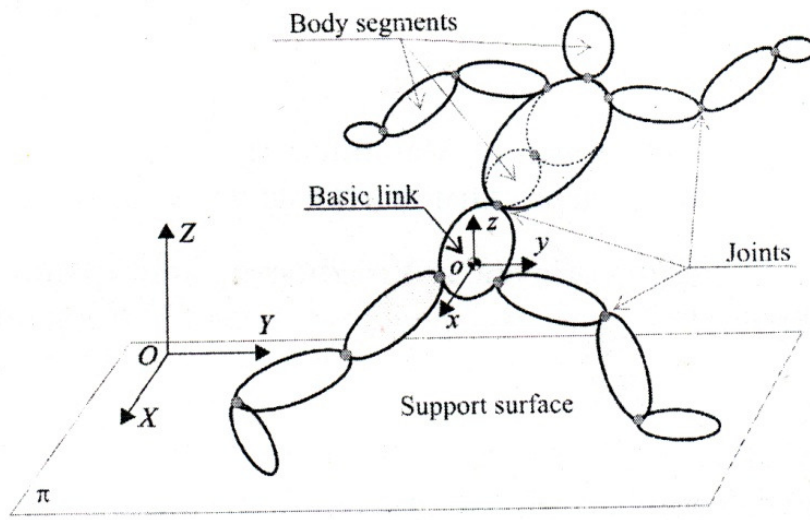
A new generalized approach to the modeling of human and humanoid motion is introduced in [3]. Generally, modeling may follow an *inductive* approach or a *deductive* one. In the

*inductive* approach, one analyzes different real situations like human or humanoid gait and running; playing tennis, soccer, or volleyball; gymnastics (exercises on the floor or by using some gymnastic apparatus); performing trampoline exercise; etc. Each problem needs a different model, appropriate to the situation – it should cover all the relevant effects. Once a number of situations are explored, one may try to make a generalization. However, there is no guarantee that it will be successful. In the *deductive* approach, one starts with considering a completely general problem. Once the general model is formulated, one may derive different real situations as being special cases. Such approach needs a serious effort to formulate a general model. This paper is an attempt in this direction to explain in detail a general modeling using deductive system.

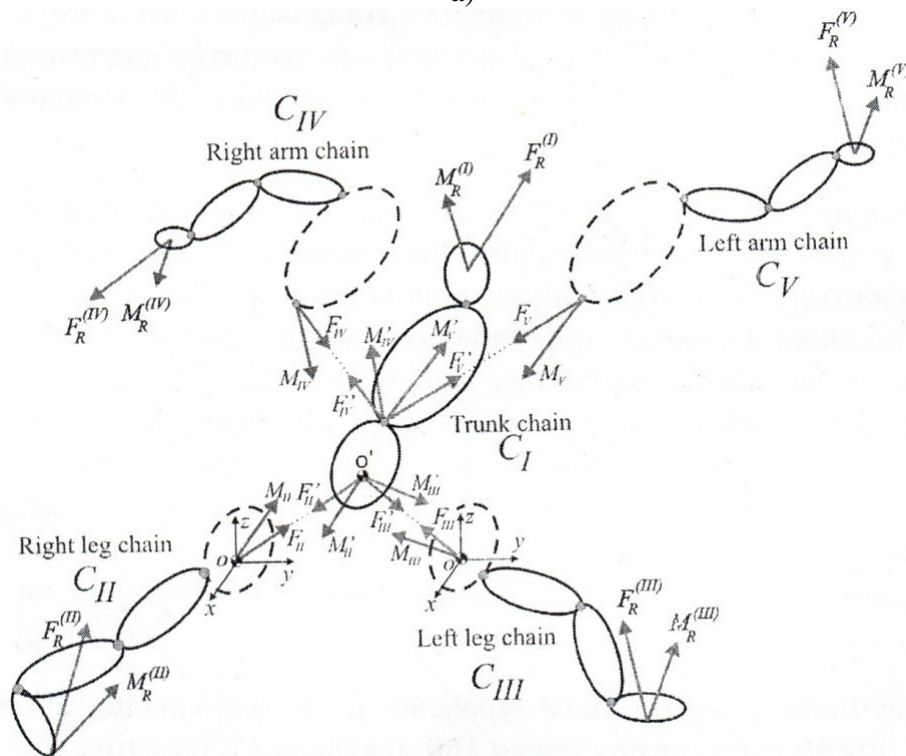
## II. MODELING OF HUMAN BODY

Human body in the mechanical sense represents a complex, multi-body branched mechanism (Fig. 1a). A body structure can be decoupled into the five kinematical chains  $C_I, \dots, C_V$  (Fig. 1b) that represent particular limbs - trunk, legs and arms. Particular kinematical chains  $C_I, \dots, C_V$  can be considered as multi-body, i.e. multi-segment kinematical mechanisms of variable structure (Fig. 2). Configuration of the particular chains is varied in real-time by moving segments in the body joints (Figs. 1a and 2). Changing configuration of kinematical chains, human body changes its kinematical (geometry) and dynamical (inertial) characteristics. As consequence of link's rotation, the system observed can be considered as system of a variable structure.

Particular mechanical chains  $C_I, \dots, C_V$  (Fig. 1b), coupled into the complex branched structure (Fig. 1a), produce mutual influence (forces and torques) to the neighbor mechanism chains (Fig. 1b). In order to develop a general spatial model of human body kinematics and dynamics, let's consider its *i-th* particular kinematical chain  $C_i$  whose general structure is presented in Fig. 2. Kinematical chain  $C_i$  has  $N_i$  degrees of freedom (d.o.f.) as presented in Fig. 2. It consists of  $j = 1, \dots, N_i$  segments/links mutually connected by corresponding rotational joints. Every segment of the mechanism  $j = 1, \dots, N_i$  (Fig. 2) has its *eigen (own) motion* as well as corresponding *transient motion* produced by displacements (rotation or translation in general case) of the previous segments in the



a)



b)

Figure 1. Kinematical structure of human body: a) coupled system of branched structure, and b) decoupled system with five multi-body subsystems

chain. Considered mechanism from Fig. 2 can be in mechanical contact with the environment or with other kinematical chains. Then we can speak about open or closed mechanisms depending on existence of such contact. External reaction forces and/or torques ( $F_R^{(i)}$  and  $M_R^{(i)}$ ) are presented in Figs. 1b and 2. These payloads of the mechanism are considered as the external reactions or as perturbation to the system. Generalized coordinates  $q_j^{(i)}$ ,  $j = 1, \dots, N_i$  determine a current position of the mechanism in 3D space. Geometry

parameters (lengths and widths of the links) and dynamic parameters (masses and inertia tensor) of the kinematical chain are known and correspond to the body measures determined experimentally with human examines. Masses of the links  $m_1, \dots, m_{N_i}$  are assumed to be situated in the particular mass centers (MC) of body links (Fig. 2). Dynamics of the coupled system (Fig. 1b) is strongly influenced by the resultant payloads produced by the particular vectors of forces

and torques  $F_i$  and  $M_i$ ,  $i = 1, \dots, V$  generated at the particular mechanical chains.

Model of the human body (i.e. mechanical system presented in Fig. 1b as its 3D model) can be defined taking into consideration fragmented mechanical system presented in Fig. 2. In that sense, corresponding characteristic vectors defining temporary position of the arbitrary link are used to derive corresponding mathematical relations. Detailed procedure of model derivation, using corresponding recursive relations, can be found in [3]. Here will be explained only basic ideas and relations leading to the coupled model of the entire branched mechanical system.

For modeling of human body the following assumptions have to be introduced:

- Human body represents a multi-body rigid system; Human limbs and body joints are not absolutely rigid. They have certain elastic characteristics. In order to simplify the modeling procedure, with no significant loose of modeling accuracy, elastic effects in the body mechanism can be neglected. There are methodologies that enable modeling of the elastic effects in the human body, too, but they are not of importance in the first stage of problem consideration.
- The first link/segment of the kinematical chain (Fig. 2) is connected to the rest of the body mechanism (other chains) by the first joint (Fig. 2). The influence of the particular mechanical chain to the other chains of human body is provided by taking into account vectors  $F_i$  and  $M_i$ .
- The last link ( $N_i$ -th link) of the chain represents an end-effector of the mechanism. Feet, hands and head are assumed to be the last links of the mechanism. Last body segments can interact (taking into the contact) with environment in general case. As the result of interaction with environment, the external reaction forces and torques appear in the MC's of these links. In that sense, we it can be spoken about the open (without contact) or closed (with contact) kinematical chains. This status is very important for modeling of the entire system since the external payloads can significantly influence body dynamics (e.g. impact forces to the ground support surface).
- Masses of the particular body links are concentrated in the corresponding MCs of body links as presented in Fig. 2.
- External forces and torques in the modeling procedure has to be reduced from contact points to the MC's of the last segments.

Generalized coordinates  $q_j^{(i)}$  (for  $i=1, \dots, V$  and  $j=1, \dots, k-1, k, k+1, \dots, N$ ;) of the mechanical system (Fig. 2) represent corresponding relative angular displacements, i.e. joint angles of the neighbor links.

Taking into account the previous assumptions, general configuration of the particular mechanism (Fig. 2) and assumed coordinate systems (the absolute one  $OXYZ$  attached to the ground support and the local one  $oxyz$  attached to the first link of the mechanism), the following vector relations can be defined.

A  $3 \times 1$  position vector of corresponding mass centre of the  $k$ -th link (Fig. 2) can be defined in the form:

$$\vec{r}_k = \vec{r}_k^o = \vec{r}_o^o + \vec{r}_{k-1}^o + \vec{r}_k^{k-1} = [X_k \ Y_k \ Z_k]^T, \quad (1)$$

where  $\vec{r}_o^o$  represents the position vector of the local coordinate system origin "o" with respect to the origin of the absolute coordinate system "O" attached to the ground support surface. Indexes "k" and "k-1" denotes corresponding order numbers of the mechanism links.

By derivation of the relation (1) the following vector equation is obtained:

$$\vec{v}_k = \frac{d\vec{r}_k}{dt} = (\vec{v}_o^o + \vec{\omega}_o \times \vec{r}_o^o + \vec{v}_{k-1}^o + \vec{\omega}_o \times \vec{r}_{k-1}^o) + (\vec{v}_k^{k-1} + \vec{\omega}_{k-1} \times \vec{r}_k^{k-1}) = [\dot{X}_k \ \dot{Y}_k \ \dot{Z}_k]^T, \quad (2)$$

where,  $\vec{v}_k$  is the  $3 \times 1$  vector of the MC velocity of the  $k$ -th mechanism link expressed in the absolute coordinate system  $OXYZ$ . This vector consist of two motion components-transition velocity  $\vec{v}_k^{TR}$  and corresponding eigen velocity  $\vec{v}_k^{EG}$ . Components of the transient and the eigen velocities are given in (3) and (4) successively.

$$\vec{v}_k^{TR} = \vec{v}_o^o + \vec{\omega}_o \times \vec{r}_o^o + \vec{v}_{k-1}^o + \vec{\omega}_o \times \vec{r}_{k-1}^o, \quad (3)$$

$$\vec{v}_k^{EG} = \vec{v}_k^{k-1} + \vec{\omega}_{k-1} \times \vec{r}_k^{k-1}, \quad (4)$$

The following notations are used:  $\vec{v}_{( )}^{( )}$  represent corresponding relative translational (linear) velocities of the points of interest with respect to other ones (e.g. "o" to "O", "k-1" to "O" or "k" to "k-1");  $\vec{\omega}_{( )}$  in general case is an angular velocity vector that determines rotation around the axis that passes through the point defined by the corresponding index (e.g. "O", "o" or mass center of the  $k-1$ -th link); It is important to stress out that the joints of a human body are rotational (but not translational) corresponding linear velocities  $\vec{v}_k^{k-1}$  in the general kinematical relations (3) and (4) are zero vectors. Due to this fact they can be left out from the corresponding relations.

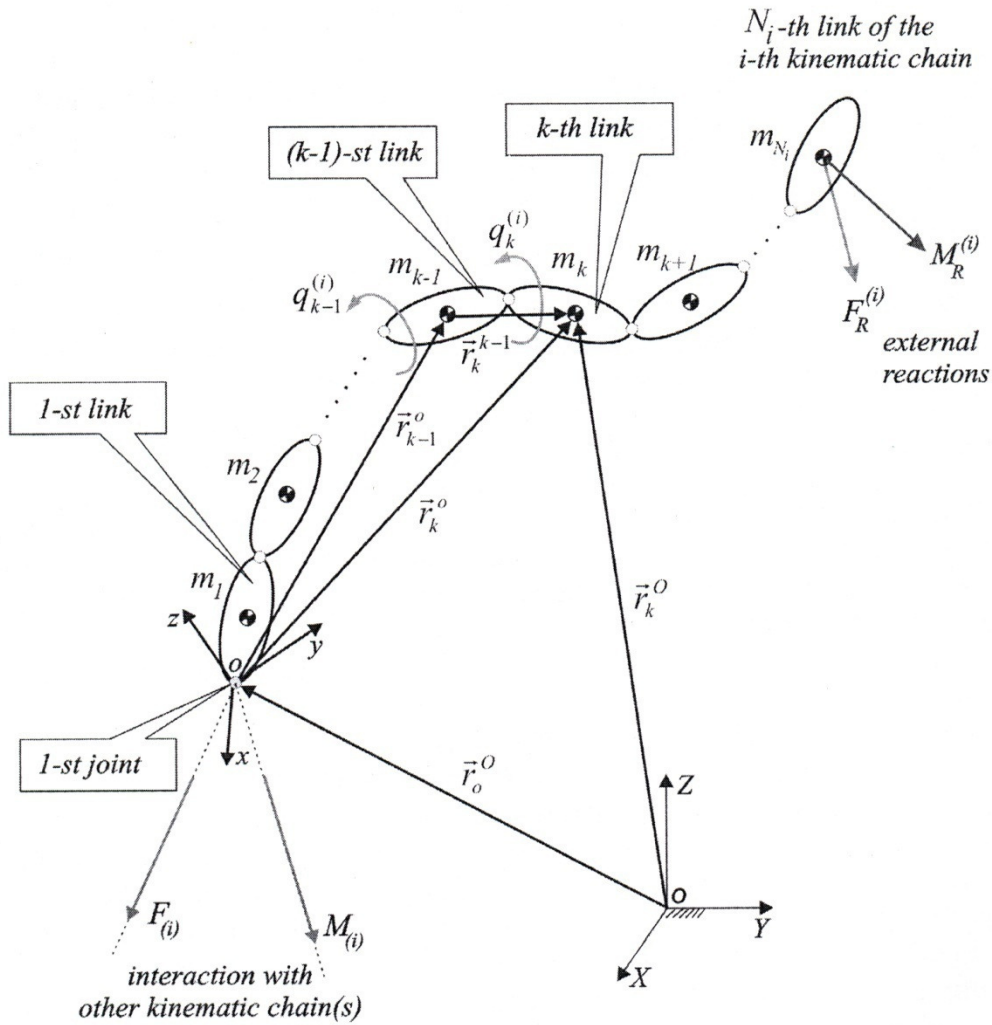


Figure 2. Particular kinematical chain  $C_i$ ,  $i = I, \dots, V$  as a part of the decoupled system (Fig. 1b)- Characteristic vectors defining mechanism position in the local  $oxyz$  and absolute  $OXYZ$  coordinate system

By derivation of (2), the equation that regards acceleration of the  $k$ -th link is obtained. Relation (5) determines vector  $\vec{a}_k$  that regards three components of acceleration of the  $k$ -th link (i.e. corresponding MC) in three coordinate directions of the absolute coordinate system  $OXYZ$ .

$$\begin{aligned} \vec{a}_k = \frac{d\vec{v}_k}{dt} = & \vec{a}_o^o + \vec{\alpha}_o \times \vec{r}_o^o + \vec{\omega}_o \times \vec{v}_o^o + \vec{a}_{k-1}^o + \\ & + \vec{\alpha}_o \times \vec{r}_{k-1}^o + \vec{\omega}_o \times \vec{v}_{k-1}^o + \vec{a}_k^{k-1} + \vec{\alpha}_{k-1} \times \vec{r}_k^{k-1} + \\ & + \vec{\omega}_{k-1} \times \vec{v}_k^{k-1} \end{aligned} \quad (5)$$

The following notation is used in (5):  $\vec{a}_o^o$ ,  $\vec{a}_{k-1}^o$  and  $\vec{a}_k^{k-1}$  are corresponding  $3 \times 1$  acceleration vectors that determine translational acceleration of points "o", "k-1" and "k" with respects to the corresponding points "O", "o" and "k-1";  $\vec{\omega}_o$ ,  $\vec{\omega}_o$ ,  $\vec{\omega}_{k-1}$  and  $\vec{\alpha}_o$ ,  $\vec{\alpha}_o$ ,  $\vec{\alpha}_{k-1}$  are  $3 \times 1$  vectors of the corresponding angular velocities and accelerations defined with respect to the axes that pass through the characteristic points "O", "o" and MC of the  $k-1$ -th link.

Relation (5) can be additionally arranged modifying the members  $\vec{\omega}_o \times \vec{v}_o^o$ ,  $\vec{\omega}_o \times \vec{v}_{k-1}^o$  and  $\vec{\omega}_{k-1} \times \vec{v}_k^{k-1}$ . The final form of the relation defining acceleration of the  $k$ -th mechanism link (Fig. 2) is obtained as:

$$\begin{aligned} \vec{a}_k = & \vec{a}_o^o + \vec{\alpha}_o \times \vec{r}_o^o + \vec{\omega}_o \times (\vec{\omega}_o \times \vec{r}_o^o) + \vec{a}_{k-1}^o + \\ & + \vec{\alpha}_o \times \vec{r}_{k-1}^o + \vec{\omega}_o \times (\vec{\omega}_o \times \vec{r}_{k-1}^o) + \vec{a}_k^{k-1} + \\ & + \vec{\alpha}_{k-1} \times \vec{r}_k^{k-1} + \vec{\omega}_{k-1} \times (\vec{\omega}_{k-1} \times \vec{r}_k^{k-1}) \end{aligned} \quad (6)$$

Now, Newton's relations for the motion of the rigid body system from Fig. 2 can be defined. Forces acting in the mass center of the  $k$ -th link of the  $i$ -th kinematical chain of human body can be expressed in the form:

$$\vec{F}_k^{(i)} = m_k^{(i)} \cdot \vec{a}_k + m_k^{(i)} \cdot \vec{g} \quad (7)$$

where  $m_k^{(i)}$  represents mass of the  $k$ -th link of the mechanism concentrated in the MC;  $\vec{a}_k$  and  $\vec{g}$  are corresponding acceleration due to the mechanism motion and corresponding

gravity acceleration. Forces acting in the particular body joints

can be calculated from the relation (8).

$$\vec{F}_j^{(i)} = \sum_{k=j}^{N_i} \vec{F}_k^{(i)} + \vec{F}_i = \sum_{k=j}^{N_i} m_k^{(i)} \cdot \vec{a}_k + \sum_{k=j}^{N_i} \vec{G}_k^{(i)} + \vec{F}_R^{(i)}, \quad j = 1, \dots, N_i, \quad k = j, \dots, N_i, \quad i = I, \dots, V \quad (8)$$

Payload in the  $j$ -th joint of the mechanical system presented in Fig. 2 is calculated as a sum of the inertial, centrifugal and Coriolis  $\sum_{k=j}^{N_i} m_k^{(i)} \cdot \vec{a}_k$ , gravitational forces  $\sum_{k=j}^{N_i} \vec{G}_k^{(i)}$  as well as corresponding external reactions  $\vec{F}_R^{(i)}$ . Corresponding joint torques  $\tau_j$ ,  $j = 1, \dots, N_i$  can be determined from the vector relation:

$$\vec{\tau}_j = \sum_{k=j}^{N_i} (\vec{r}_j^k \times \vec{F}_k^{(i)}) + \vec{r}_{N_i}^j \times \vec{F}_R^{(i)} + \vec{M}_R^{(i)}, \quad j = 1, \dots, N_i, \quad i = I, \dots, V \quad (9)$$

For the first joint of the mechanism the following relations are valid:

$$\vec{F}_i = \vec{F}_j^{(i)}, \quad j = 1 \quad (10)$$

$$\vec{M}_i = \vec{\tau}_j^{(i)}$$

Now, the influence of the particular kinematical chain " $i$ " (Fig. 1b and 2) to the rest of human body mechanism can be taken into account through the following relations:

$$\vec{F}'_i = -\vec{F}_i, \quad i = I, \dots, V \quad (11)$$

$$\vec{M}'_i = -\vec{M}_i$$

In such a way, the relation (11) enables coupling of dynamics of the particular kinematical chains  $i = I, \dots, V$  to the dynamics of the overall system - human body. Relation (1) to (11) describes general relations for building a mathematical model of human body dynamics with arbitrary degrees of freedom. Taking into account all d.o.f.-s of human body, it is possible to write the entire, coupled model of the mechanical system presented in Fig. 1b in the following matrix form [5, 6]:

$$\tau = H(Q, d)\ddot{Q} + h(Q, \dot{Q}, d) - J_1^T(q_{(I)}, d)\phi_R^{(I)} - \dots - J_V^T(q_{(V)}, d)\phi_R^{(V)}, \quad \tau \in \mathfrak{R}^{N \times 1}$$

$$Q = [q_{(I)}, q_{(II)}, \dots, q_{(V)}]^T, \quad Q \in \mathfrak{R}^{N \times 1}, \quad N = \sum_{i=1}^V N_i$$

$$H(Q, d) = \{H_{i,j}\}, \quad i = 1, \dots, 5, \quad k = 1, \dots, N_i, \quad H \in \mathfrak{R}^{N \times N}$$

$$h(Q, \dot{Q}, d) \in \mathfrak{R}^{N \times 1}$$

$$d \in \mathfrak{R}^{N \times 1} \quad (12)$$

$$\phi_i = [F_{Rx}^{(i)}, F_{Ry}^{(i)}, F_{Rz}^{(i)}, M_{Rx}^{(i)}, M_{Ry}^{(i)}, M_{Rz}^{(i)}]^T, \quad \phi_i \in \mathfrak{R}^{6 \times 1}, \quad i = I, \dots, V$$

$$J_i = J_i(q_{(i)}, d), \quad J_i \in \mathfrak{R}^{6 \times N_i}$$

Where the following notation is used:  $\tau$  is the vector of the joint torques of human body mechanical system;  $Q$  is the corresponding vector of the generalized coordinates (joint angles);  $H(\cdot)$  is an inertia matrix calculated by coupling inertia matrices of the particular mechanical chains;  $h(\cdot)$  is a coupled vector of the centrifugal, Coriolis and gravitational torques acting in the body joints;  $d$  is a vector of body parameters (geometry, kinematical and dynamical);  $\phi_i$  is the vector of the external reactions forces and torques acting to the last segment of the  $i$ -th particular mechanism;  $J_i$  is the Jacobian matrix of the  $i$ -th particular kinematical chain based on the relation (1) and its functional dependency on generalized coordinates  $q_{(i)}$  [3, 4].

### III. CONCLUSION

In this paper it was intended to apply sophisticated methods developed for mathematical modeling of humanoid robots to real human motions. A general simulation system is already realized: following a deductive principle the algorithm considers particular human/humanoid motions (like those occurring in different sports) as being just special cases of a general motion and impact theory. Here the modeling of human body is presented using general model approach. Several human-like movements has been realized [6, 7].

### ACKNOWLEDGMENT

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