NUMERIČKA SIMULACIJA 3D FOTONIČKIH TALASOVODNIH STRUKTURA PROIZVOLJNOG OBLIKA

NUMERICAL SIMULATION OF 3D ARBITRARY SHAPED PHOTONIC WAVEGUDE STRUCTURES

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Sadržaj – J*edan od najčešće korišćenih numeričkih alata za simulaciju fotoničkih struktura zasnovanih na dielektričnim talasovodima je tzv. "beam propagation method (BPM)". U ovom radu posmatrana je varijanta BPM zasnovana na metodi konačnih razlika i na transformaciji koordinatnih sistema, tzv. "structure related finite difference beam propagation method (SR-FD-BPM)". U suštini, ova nova simulaciona tehnika u optoelektronici je jednostavna, fleksibilna i idealna za primenu u CAD okruženju, posebno u analizi prostorno zakrivljenih talasovodnih struktura. Simulacione metode zasnovane na transformaciji koordinatnih sistema, kao npr. "structure-related FD-BPM", omogućavaju uspešno CAD projektovanje geometrijski složenih fotoničkih komponenti, koje se danas intenzivno koriste u proizvodnji fotoničkih čipova. Dati su i ilustrativni rezultati SR-FD-BPM simulacija.*

Abstract – *One of the most widely used numerical simulation tools for modeling waveguide-based photonic structures is the beam propagation method (BPM). In this paper the attention is paid on the structure related finite difference beam propagation method (SR-FD-BPM). This recently developed simulation technique for optoelectronic design is conceptually straightforward, flexible and ideally suited for CAD software, especially when the waveguide structure under analysis is changing in the direction of the propagation, containing oblique or curved interfaces. Coordinate transformation approaches, such as structure-related FD-BPM, allow the comfortable analyze of wide variety of geometrically complex light-wired photonic structures witch are used frequently in photonics circuitry CAD design. Some illustrative simulation results, based on an efficient SR-BPM algorithm, are presented in the paper.*

Keywords – *Beam propagation method, Finite difference beam propagation method, Structure related beam propagation method, Numerical simulation, Helmholtz Equation, Photonics, Optoelectronic design, CAD software.*

1. INTRODUCTION

The recent years enormous increase in bandwidth requirements for telecom and datacom transmission systems, including in large part the growth of the Internet, have put photonics and integrated optics in a focus of the current electronics industry. The light-wave transmission systems, photonic integrated circuits (PIC) and fiber-optic photonic components are the foundation of such systems. Continual and stringent demands on the photonics community are to develop the new or improved technologies enabling production of these components [fibers, lasers, detectors, modulators, couplers, switches, wavelength-demultiplexing devices (WDM's), etc.].

Behind the industrial and commercial scene, however with crucial importance have been the research and the developments in modeling techniques and introduction of computer-aided design (CAD) software for modeling opto-electronic components and systems. Design methodologies and tools play extremely significant roles in the advancement of optical components. Commercial and in-house tools would not be possible without the development in modeling techniques for optoelectronic and photonics. Ever-increasing demands to enable telecom and datacom system to meet very stringent requirements significantly rely on research and advances in modeling techniques.

Basically, optical modes can propagate in a given, uniform cross-section of a wave guiding structure. Most photonic devices, however, change their shape in the direction of propagation. In such cases, technique that can handle this serious design issue is the beam propagation method (BPM). The BPM is, certainly, the most widely used propagation technique for modeling photonic devices, and most commercial software for such modeling is based on it. BPM is essentially a particular approach for numerical solving of appropriate approximation of the exact wave equation for monochromatic waves. Amongst several BPM algorithms developed in recent two decades (FE-BPM, MoL-BPM, FDTD-BPM, etc.), one of the most commonly used simulation algorithm in integrated optics is the frequency-domain based finite difference beam propagation method (FD-BPM), [1-3].

The standard implementation of FD-BPM in a rectangular coordinate system causes serious problems and certain restrictions if the structure under analysis contains oblique or curved interfaces or when the structure is changing in the direction of the propagation. The source of these restrictions is the inevitable staircase approximation of the boundaries which occurs during the finite difference discretization procedure. This deficiency has been successfully remedied for special cases by the coordinate transformation approaches, [4-8]. To avoid using the fine meshes and small propagation steps, FD-BPM has been reformulated in non-orthogonal, structure related (SR), coordinate systems, [9-12]. Another promising approach has been recently proposed in [13-14], featuring the separation of the BPM propagation algorithm from the discretization FD grid.

The goal of this paper is to review some advances in the recently developed structure related beam propagation methodology. The author has been a member of the Nottingham's Electromagnetics Research Group, today the George Green's Institute, for several years, contributing to those major advances in photonics, [2,3,6,7,10,11,12].

2. STRUCTURE RELATED FD-BPM

Many attempts have been made during the last two decades to overcome the presence of the staircasing in the finite difference related numerical methods. These have been resulted in the developing of the so-called improved FD-BPM schemes in rectangular coordinate environment for step index devices, [15]. The improved FD approach takes into account the boundary conditions for the field and its derivatives near the dielectric interfaces, which results to second order accurate formulas for the second derivatives of the field. However, in the case of the curved waveguide structures in the propagation direction using the standard rectangular coordinate system, it is obvious that certain coefficients in improved discretization formulas have to be calculated before every forward z-step. Consequently, such algorithm accumulates numerical noise during the propagation with the increasing of the run-time, especially for fine meshes and smaller propagation steps.

The Nottingham's research team have pioneered the use of structure related coordinate schemes for FD-BPM, where the discetrisation procedure exactly matches the local geometry of the structure, thus eliminating non-physical scattering due to the staircasing effect. The SR coordinate system approach allows only one initial computation of the improved FD formulas coefficients of the same order of the accuracy during the propagation. The resulting SR FD-BPM (SR-BPM) algorithm allows simulations with noticeably reduced numerical noise and shortened simulation time. The advantage of this approach is that, in comparison to the standard rectangular schemes, coarser mesh sizes can be used for the same accuracy offering a significant reduction in computational time involved, particularly in 3D simulations.

SR-BPM approach and resulting algorithm is used nowadays to design tapered, oblique and bi-oblique shaped optoelectronic waveguides and waveguide-based devices, such as z-variant directional waveguide couplers, y-branches, optical interconnects, waveguide polarizers and similar photonics integrated circuits (PIC) components that include waveguide bends. Some illustrative examples and results are given in present paper, involving a paraxial, semi-vectorial 3D SR-BPM analysis of "S"-curved directional waveguide couplers, together with comparison with results from a similar standard analysis using a rectangular coordinate system. Comparison of the results for the same order of discretization readily shows the distinct advantages of SR schemes and SR-BPM algorithm: the propagation error is reduced; the simulation time is shortened, particularly in 3D simulations which are the practical reality in optoelectronics.

3. THEORETICAL BASICS OF SR-BPM

In this section, the basic approach of SR-BPM is illustrated by formulating scalar and semi-vectorial (polarized) SR-BPM wave-equation. Under the assumption that the refractive index $n(x, y, z)$ at operating wavelength λ_0 varies slowly along the propagation direction z, one can derive the so-called semi-vectorial Helmholtz wave equation based on the transverse electric field $E_t = E_t(x, y, z)$, or transverse magnetic field $H_t = H_t(x, y, z)$,

$$\nabla^2\Phi_t + n^2k^2\Phi_t = 0, \tag{1}$$

where k is the local wave number, $k = k_0 n$, $k_0 = 2\pi/\lambda$ is the free space wave number, Φ_t can be either E_t or H_t. If, in the general 3D non-orthogonal coordinate system (u, v, w) we choose the coordinates (u, w) as $x = f(u, w)$, $z = w$ and $y = v$, we obtain the SR wave equation, [9,11],

$$\left[A\frac{\partial^2}{\partial u^2} - B\frac{\partial^2}{\partial w\,\partial u} + C\left(k^2 + \frac{\partial^2}{\partial w^2}\right) - D\frac{\partial}{\partial u} + \frac{\partial^2}{\partial y^2}\right]\Phi_t = 0, \tag{2}$$

where $\Phi_t = \Phi_t(u, y, w)$ can be either $E_t = E_t(u, y, w)$ or $H_t = H_t(u, y, w)$. The first and second derivatives in (2) with respect to non-orthogonal transverse plane coordinate u and second derivatives with respect to standard coordinate y are considered to be discontinuous at the boundaries, and $A = A(u, w)$, $B = B(u, w)$, $C = C(u, w)$, $D = D(u, w)$ are functions of the partial derivatives of $f(u, w)$, [9],

$$A = 1 + \left(\frac{\partial f}{\partial w}\right)^2, \quad B = 2\left(\frac{\partial f}{\partial u}\right)\left(\frac{\partial f}{\partial w}\right), \quad C = \left(\frac{\partial f}{\partial u}\right)^2,$$

$$D = \frac{1}{\frac{\partial f}{\partial u}}\left[A\frac{\partial^2 f}{\partial u^2} - B\frac{\partial^2 f}{\partial w\,\partial u} + C\frac{\partial^2 f}{\partial w^2}\right].$$

The propagation is assumed to be in the $+w$ (i.e. the $+z$ with standard rectangular coordinate) direction and the field is separated as a slowly-varying envelope function F_t and a fast-oscillating exponential phase term,

$$\Phi_t(u, v, w) = F_t(u, y, w)e^{-j\beta w}, \tag{3}$$

with β an imposed background propagation constant which has to be determined. By substituting of (3) in (2) and ignoring the $\partial^2/\partial w^2$ term, the paraxial one-way wave equation in the 3D non-orthogonal coordinate system (u, y, w) can be derived as,

$$L\frac{\partial}{\partial w}F_l = MF_l\,, \tag{4}$$

where the operators L and M are shown to be, [9],

$$L = 2j\beta C + B\frac{\partial}{\partial u}, \tag{5}$$

$$M = A\frac{\partial^2}{\partial u^2} + (Bj\beta - D)\frac{\partial}{\partial u} + C(k^2 - \beta^2) + \frac{\partial^2}{\partial y^2}. \tag{6}$$

Equations (2-4) are derived under the scalar approximation of the field, but they can be straightforwardly upgraded to a semi-vectorial BPM algorithm. Semi-vectorial SR approach can be applied for desired function $x = f(u, w)$ assuming that propagation occurs in the w direction.

A standard Crank-Nicolson method is easily introduced in the 3D SR-BPM algorithm. For the well confined waveguide fields transparent boundary conditions (TBC), [16], are typically used at the edges of the computational window, otherwise the Berenger's perfectly matched layers (PML) have to be introduced in the algorithm.

3. SIMULATION RESULTS

FD-BPM methods have been extensively used in analysis of bent and curved waveguide based photonics devices. The SR approach allow designer the flexibility and comfortable analysis of not only the circular-like bends and curvatures, designer has freedom of choice to use any curvature function providing optimal integrated optic requirements, such as to achieve compact low loss of optoelectronic circuits.

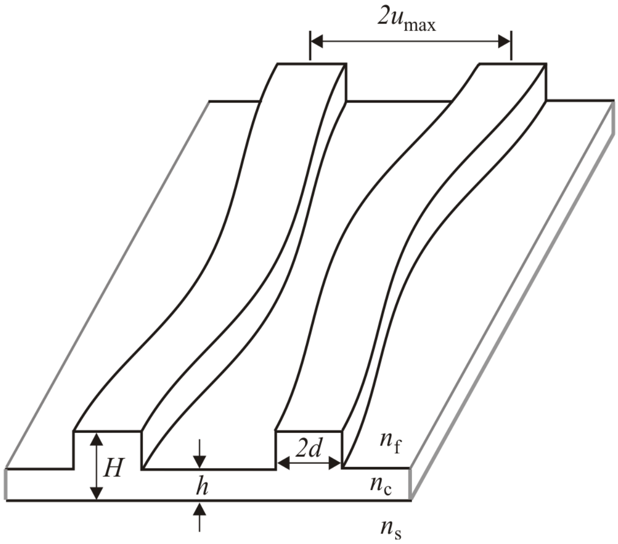

Fig. 1. Geometry of 3D directional waveguide coupler in SR coordinate system, (u, w) plane.

Some illustrative examples of 3D curved directional couplers design are presented to highlight the effectiveness and flexibility of the SR FD-BPM. The propagation of the fundamental TE and TM modes, with both E- and H-field formulations, are studied and results compared with those obtained from simulations based on a rectangular coordinate system. The method can be straightforwardly applied to single curved waveguide cases or even more complex structures with curved sections.

Waveguide directional couplers exchange the power between guided modes of adjacent waveguides and perform a number of useful functions in optoelectronic circuits, such as power division, switching and modulation. The "S" curved couplers analysed have advantage over couplers produced from straight segments because of their lower loss properties. Figure 1 shows a symmetrical coupler made from two identical and adjacent but spatially separated curved input and output waveguides and it is used for total power transfer from the input to the output guide. The asymmetrical coupler configuration is shown respectively in Figure 2. In general, one or more curved coupled waveguides with different curvatures and different cross-sections can be considered.

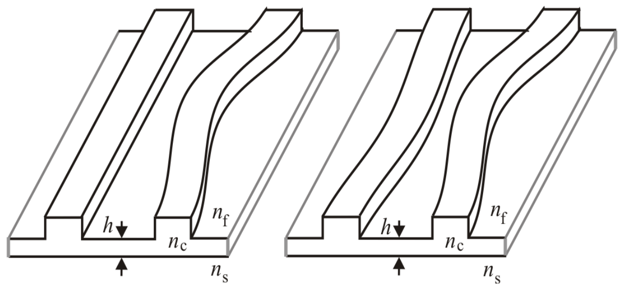

Figure 2. The asymmetrical waveguide coupler diagrams. The coupler is modeled with straight and curved guide (on the left), or with guides of different curvatures (on the right).

The curve function of a "S" curved coupler can be given parametrically. Amongst a various possibilities, because of the simplicity, in the proceeding simulation example a cosine type structure related geometry was considered (for $x > 0$; symmetrically for $x < 0$),

$$x = f(u, v) = u - U_m\left(1 - \cos\frac{2\pi w}{L_c}\right), \tag{7}$$

for $u \geq u_0$, and

$$x = f(u, v) = \frac{u}{u_0}\left[u_0 - U_m\left(1 - \cos\frac{2\pi w}{L_c}\right)\right], \tag{8}$$

for $u \leq u_0$, where $U_m = (u_{max} - u_{min})/2$, coupler configuration and geometry shown in Figure 3. Functions A, B, C and D can be easily obtained analytically, or can be even computed numerically. If the curvature function changes slowly with w, only the function C varies as the square of the gradient of $f(u, w)$ with respect to the oblique coordinate u, and if the condition $(w_{max} - w_{min}) \ll L_c$ is fulfilled, we can introduce $A \simeq 1, B \simeq 0, D \simeq 0$, which is always the case except for the sharp guide bends. The discretization mesh, in the (u, z) plane, Figure 3, in the y direction is performed in the standard rectangular manner. The meshing in the u direction follows the curved geometry of waveguides. Around the waveguides the u=const. lines are 'parallel' to the dielectric boundaries enabling the efficient equally distributed discretization. In the region between the guides, $-u_0 > u > +u_0$, the mesh density can be performed with noticeably high degree of freedom, with no significant influence on the accuracy of the simulation results.

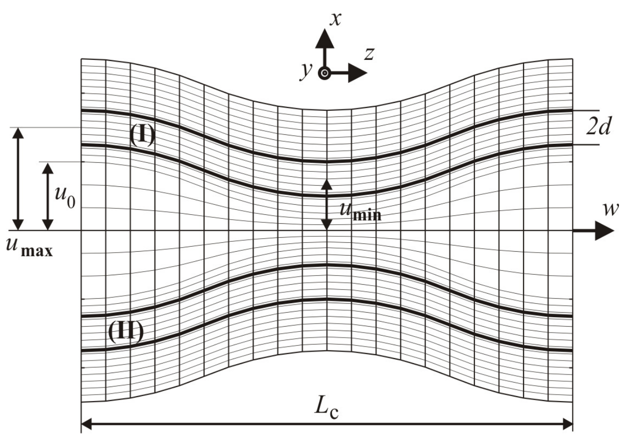

Figure 3. Configuration and geometry of 3D symmetrical rib waveguide coupler in SR coordinate system, (u, w) plane.

In 3D simulations performed, the waveguide (I) is considered as input and waveguide (II) as output guide, which can be interchanged because of the symmetry. At $z = 0$ the TE or TM mode field is launched from the accurate mode solver in the waveguide (I). The geometry of the coupler guides is kept constant. The total coupling length L_c has to be determined with numerical simulations, under the condition of total power transfer of guided fundamental TE or TM mode from the input (I) to the output (II) mode. The total coupling length L_c is calculated with both rectangular and SR based FD-BPM algorithms, with standard rectangular one using a very fine mesh and small steps. Both algorithms tend to give the same answer for L_c. The numerical accuracy of the standard and SR-BPM simulation is evaluated in terms of mode mismatch loss L_M, defined in the 3D case as

$$L_M = -10 \log \frac{\left| \int E_i E_c^* \, du \, dy \right|^2}{\left[\int |E_i|^2 \, du \, dy \right]^2} \quad [dB], \qquad (9)$$

where E_i is the incident fundamental mode at $w = 0$ within guide (I) and E_c is the coupled propagation field in guide (II) at $w = L_c$ obtained using the BPM simulation.

A 3D symmetrical rib waveguide coupler of type (7-8), see the inset in Figure 4, and with the (u, w) plane defined as in Figure 3, has been numerically analyzed, and gathered data readily indicate the efficiency and accuracy of the SR approach. Rib waveguides have the identical cross-sections with $n_f = 1$, $n_c = 3.44$, $n_s = 3.4$, $H = 1\mu$m, $h = 0.5\mu$m, $2d = 3\mu$m, $\lambda = 1.15\mu$m,. The total coupling of the fundamental TE mode $(n_{ref_{TE}} = 3.41313)$ is obtained with $u_{max} = 3.0\mu$m, $u_{min} = 1.8\mu$m and the total coupling length $L_{c_{TE}} = 4826\mu$m. For TM-mode $L_{c_{TM}} = 4769\mu$m is obtained. The L_M error in TM-mode propagation as function of the transverse mesh sampling is shown in Figure 4. It is obvious again that the SR scheme enables more accurate 3D simulations of the total coupling. In the SR scheme the number of u coordinate lines in the coupling region can be relaxed, the index mapping and FD improved formulas coefficients are calculated only once at $z = 0$, consequently giving considerably shorter simulation time for the same order of accuracy in comparison to the rectangular scheme. Both TE- and TM-mode propagation algorithms exhibit almost identical behaviour in terms of accuracy, only the total coupling lengths are different, which is due to the different background propagation constants.

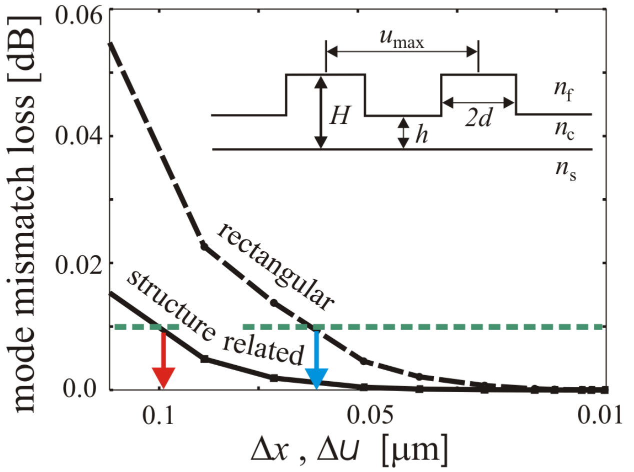

Figure 4. Mode mismatch loss at $w = L_c$ versus mesh size, TM propagation, for the 3D coupler in inset of figure.

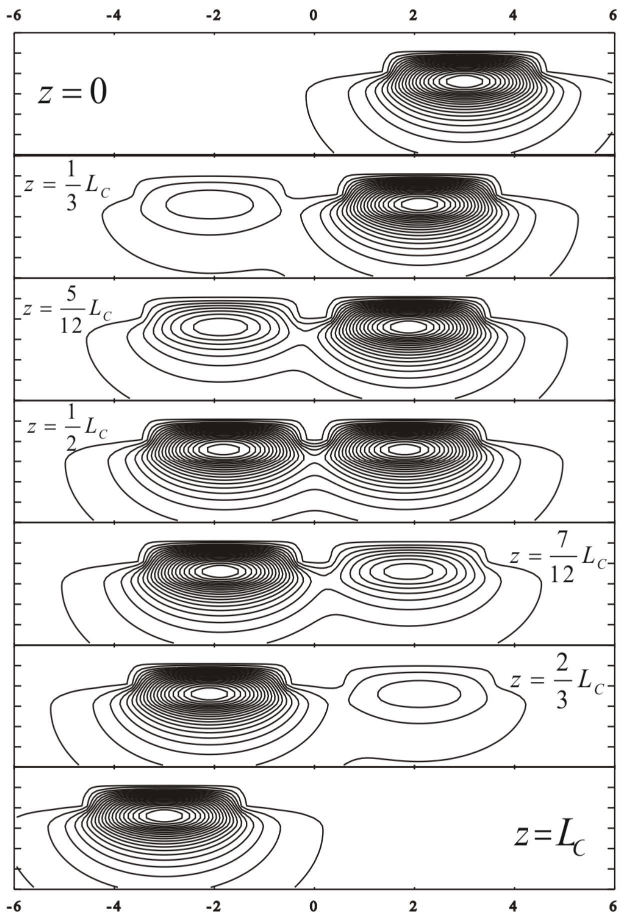

Figure 5. The evolution of the total power transfer of the TE mode H-field during the propagation in 3D coupler.

The Figure 5 shows the total power coupling of the fundamental TE mode H-field within the symmetrical coupler. The transverse mesh size was kept to be $\triangle x = \triangle y = 0.1$ μm with 600 steps in the w propagation direction. The outer H-field contour in Figure 5 is just 1% of maximal coupler field, the close one is 5% of the maximal field, and others show 10% field increments. The H-fields in every w-slice are normalised to the maximum field. It is obvious from Figure 5, that the almost complete power transfer occurs in the coupler region where rib waveguides are closest to each other.

Due to unconditionally stable Crank-Nicolson procedure which was used in the algorithm, longitudinal step size $\triangle w$ do not affect significantly the accuracy of the simulation. In 3D example described above, while the transverse mesh size is of the order 0.1 μm and less, very accurate simulations have been obtained with $\triangle w$ of the order 10 μm. Also, various parametric curves were used in discretizing the region between two coupled guides, with uniform and non-uniform spacing. Analysis of the results show that the accuracy of simulations is affected significantly, measured in terms of mode mismatch loss L_M, only when the grid redistribution is performed near boundaries. This leads to the conclusion that the accuracy of the BPM simulations depends strongly and mostly of the method used for the discretization of the boundaries region of the waveguide.

The vectorial SR BPM version of the algorithm is time-consuming, however, results obtained for the coupling length L_c do not differ significantly from those obtained for the same meshes and under the same parameters as in simulations under the semi-vectorial approximation. This leads to the conclusion that in the analysis of the coupling in the curved but with rectangular cross-section waveguide couplers within a millimetre coupling length range a full-vectorial approach is not mandatory.

5. CONCLUSION

In this paper, some advances in the structure related beam propagation method have been addressed and reviewed. The main feature of the SR based method is an exact modelling of the local structure geometry. The numerical simulations, based on an efficient structure related FD-BPM algorithm, have been carried out to analyse propagation in 3D curved directional couplers. Results, compared with those obtained from simulations based on a rectangular coordinate system, demonstrate the advantages and generality of structure related over standard rectangular approach. The SR-BPM method offers flexibility in design of geometrically complex structures with curved sections, and significant computational resource savings.

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