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PROBABILITY DENSITY FUNCTION OF THE SATTELITE SIGNAL IN THE PRESENCE OF RAYLEIGH FADING ON SATTELITE AND EARTH STATION

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Abstract –*In this paper satellite communication system consisting of the earth transmitting station and the satellite transponder is considered. SSC diversity technics are used on receiving satellite and receiving earth stations to reduce fading influence to the system performances. The presence of Rayleigh fading on receiving satellite and receiving earth stations is observed. The probability density functions (PDFs) of the signal at the Earth receiver station output are given for different parameters.*

1. INTRODUCTION

Satellite communication systems are now a major part of most telecommunications networks as well as every-day lives through mobile personal communication systems and broadcast television [1]. A fundamental understanding of such systems is therefore important for a wide range of system designers, engineers and users.

The fading and shadow effect are factors which degrade the system performances in telecommunication systems at the most. They derogate the power of transmitted signal. When a received signal experiences shadow effect or fading during transmission, its envelope and phase fluctuate over time. The most often are Rayleigh, Rice, Nakagami, Weibull and log-normal fading, and they are considered in the literature [2], [3].

Rayleigh fading is fading in a satellite communications channel due to the interference caused to the main signal by the same signal arriving over many different paths, resulting in out-of-phase components incident at the receiver. Rayleigh fading occurs commonly in wireless communications channels, including satellite communications channels.

In wireless communication systems various techniques for reducing fading effect and influence of shadow effect are used. Such techniques are diversity reception, dynamic channel allocation and power control. Upgrading transmission reliability and increasing channel capacity without increasing transmission power and bandwidth is the main goal of diversity techniques.

Diversity reception, based on using multiple antennas at the receiver, (space diversity, with two or more branches), is very efficient methods used for improving system's quality of service, so it provides efficient solution for reduction of signal level fluctuations in fading channels. Multiple received copies of signal could be combined on various ways.

Several principal types of combining techniques can be generally performed by their dependence on complexity restriction put on the communication system and amount of channel state information available at the receiver. Combining techniques like maximal ratio combining (MRC) and equal gain combining (EGC) require all or some of the amount of the channel state information of received signal. Second, MRC and EGC require separate receiver chain for each branch of the diversity system, which increase its complexity of system.

Maximal-Ratio Combining (MRC) is one of the most widely used diversity combining schemes whose SNR is the sum of the SNR's of each individual diversity branches. MRC is the optimal combining scheme, but its price and complexity are higher. Also, MRC requires cognition of all channel parameters and admit in the same phase all input signals, because it is the most complicated for realization ([4]-[6]).

Signal at the EGC diversity system output is equal to the sum of its' input signals. The input signals should be admitted in the same phase, but it is not necessary to know the channel parameters. Therefore, EGC provides comparable performances to MRC technique, but has lower implementation complexity, so it is an intermediate solution [7].

One of the least complicated combining methods is selection combining (SC). In opposition to previous combining techniques, SC receiver processes only one of the diversity branches, and it is much simpler for practical realization. Generally, selection combining, selects the branch with the highest signal-to-noise ratio (SNR), that is the branch with the strongest signal [7], [8], assuming that noise power is equally distributed over branches. Similarly to previous, there is type of selection combining that chooses the branch with highest signal and noise sum. In fading environments where the level of the cochannel interference is sufficiently high comparing with the thermal noise, SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [9].

SSC is an attempt at simplifying the complexity of the system but with loss in performance. In this case, rather than continually connecting the antenna with the best fading conditions, the receiver selects a particular antenna until its quality drops below a predetermined threshold. When this happens, the receiver switches to another antenna and stays with it for the next time slot, regardless of whether or not the channel quality of that antenna is above or below the predetermined threshold. The consideration of SSC systems in the literature has been restricted to low-complexity mobile units where the number of diversity antennas is typically limited to two ([10]-[12]). Furthermore, in all these publications, only predetection SSC has thus far been considered wherein the switching of the receiver between the two receiving antennas is based on a comparison of the instantaneous SNR of the connected antenna with a predetermined threshold. This results in a reduction in complexity relative to SC in that the simultaneous and continuous monitoring of both branches SNRs is no longer necessary.

MRC dual diversity systems in the presence of Rayleigh and log-normal fading are analyzed in [13], [14] and SSC combiner output signal in the presence of Rayleigh Fading in [15], [16]. We analyse Rayleigh fading influence to the satellite telecommunication system performances in this paper.

2. SYSTEM MODEL

The use of SSC combiner with great number of branches can minimize the bit error rate (BER). We determined SSC combiner with two inputs because the gain is the greatest when we use the SSC combiner with two inputs instead of one-channel system. When we enlarge the number of inputs (branches) the gain becomes less. Because of that it is more economic using SSC combiner with two inputs.

Let see how the SSC combiner with two inputs works. The model of this system is shown in Figure 1.

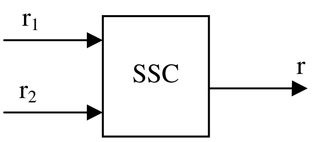

Fig.1. Model of the SSC combiner with two inputs

The signals at the combiner input are r_1 and r_2, and r is the combiner output signal. The probability of the event that the combiner first examines the signal at the first input is P_1, and for the second input is P_2. If the combiner examines first the signal at the first input and if the value of the signal at the first input is above the treshold, r_T, SSC combiner forwards this signal to the circuit for the decision. If the value of the signal at the first input is below the treshold r_T, SSC combiner forwards the signal from the other input to the circuit for the decision.

If the SSC combiner first examines the signal from the second combiner input it works in the similar way.

There are two diversity branches on satellite and on Earth receiver station. The SSC combining is used on both, receiver satellite and Earth station. System model is shown in the Figure 2.

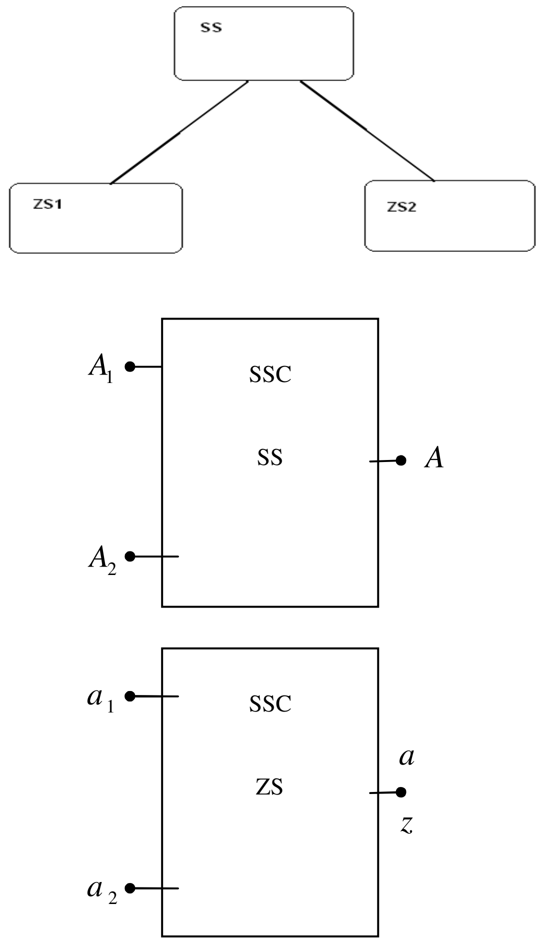

Fig.2. System model

Let Rayleigh Fading be present on both, receiver Satellite and Earth Station. In this case the probability density of signal A at the Satellite station output is, for $0<A<A_T$:

$$p_A(A) = P_{1A} p_{A_2}(A) F_{A1}(A_T) + P_{2A} p_{A_1}(A) F_{A2}(A_T) \quad (1)$$

and for $A_T<A$

$$p_A(A) = P_{1A} p_{A_1}(A) + P_{1A} p_{A_2}(A) F_{A1}(A_T) + $$
$$+ P_{2A} p_{A_2}(A) + P_{2A} p_{A_1}(A) F_{A2}(A_T) \quad (2)$$

The probability density of signal a at the Earth receiver station output is, for $0<a<a_T$:

$$p_a(a) = P_{1a} p_{a_2}(a) F_{a1}(a_T) + P_{2a} p_{a_1}(a) F_{a2}(a_T) \quad (3)$$

and for $a_T<a$:

$$p_a(a) = P_{1a} p_{a_1}(a) + P_{1a} p_{a_2}(a) F_{a1}(a_T) +$$

$$+P_{2a}p_{a_2}(a)+P_{2a}p_{a_1}(a)F_{a2}(a_T) \quad (4)$$

The signal z at the Earth receiver station output is:

$$z = a\cos\varphi + y_1 = a\frac{A+x_1}{\sqrt{(A+x_1)^2+y_1^2}} + y_1 \quad (5)$$

The conditional probability density of the signal z is:

$$p_z(z/x_1,y_1,A,a) = \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{[z-f_1(a,A,x_1,y_1)]^2}{2\sigma_2^2}} \quad (6)$$

The probability density of the signal z is:

$$p_z(z) = \int dx_1 \int dy_1 \int da \int dA \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{[z-f_1(a,A,x_1,y_1)]^2}{2\sigma_2^2}} \cdot$$

$$\cdot p_A(A)\, p_a(a)\, p_{x_1y_1}(x_1,y_1) \quad (7)$$

The joint probability density of the Gaussian components $x_1$and x_2 is:

$$p_{x_1y_1}(x_1,y_1) = \frac{1}{2\pi\sigma_1^2} e^{-\frac{x_1^2+y_1^2}{2\sigma_1^2}} \quad (8)$$

The function $f_1(a,A,x_1,y_1)$ is defined with:

$$f_1(a,A,x_1,y_1) = a\frac{A+x_1}{\sqrt{(A+x_1)^2+y_1^2}} \quad (9)$$

The probability density of output signal z, after some substitutions, is:

$$p_z(z) = \int_{-\infty}^{+\infty} dx_1 \int_{-\infty}^{+\infty} dy_1 \int_0^{+\infty} da \int_0^{+\infty} dA \frac{1}{\sqrt{2\pi}\sigma_2} \cdot$$

$$\cdot e^{-\frac{[z-f_1(a,A,x_1,y_1)]^2}{2\sigma_2^2}} \frac{1}{2\pi\sigma_1^2} e^{-\frac{x_1^2+y_1^2}{2\sigma_1^2}} p_a(a)p_A(A) =$$

$$= \int_{-\infty}^{+\infty} dx_1 \int_{-\infty}^{+\infty} dy_1 \int_0^{a_T} da \int_0^{A_T} dA \frac{1}{\sqrt{2\pi}\sigma_2} \cdot$$

$$\cdot e^{-\frac{[z-f_1(a,A,x_1,y_1)]^2}{2\sigma_2^2}} \frac{1}{2\pi\sigma_1^2} e^{-\frac{x_1^2+y_1^2}{2\sigma_1^2}} .$$

$$\cdot [P_{1A}p_{A_2}(A)F_{A1}(A_T)+P_{2A}p_{A_1}(A)F_{A2}(A_T)] \cdot$$

$$\cdot [P_{1a}p_{a_2}(a)F_{a1}(a_T)+P_{2a}p_{a_1}(a)F_{a2}(a_T)] +$$

$$+ \int_{-\infty}^{+\infty} dx_1 \int_{-\infty}^{+\infty} dy_1 \int_{a_T}^{+\infty} da \int_0^{A_T} dA \frac{1}{\sqrt{2\pi}\sigma_2} \cdot$$

$$\cdot e^{-\frac{[z-f_1(a,A,x_1,y_1)]^2}{2\sigma_2^2}} \frac{1}{2\pi\sigma_1^2} e^{-\frac{x_1^2+y_1^2}{2\sigma_1^2}} .$$

$$\cdot [P_{1A}p_{A_2}(A)F_{A1}(A_T)+P_{2A}p_{A_1}(A)F_{A2}(A_T)] \cdot$$

$$\cdot [P_{1a}p_{a_1}(a)+P_{1a}p_{a_2}(a)F_{a1}(a_T) +$$

$$+P_{2a}p_{a_2}(a)+P_{2a}p_{a_1}(a)F_{a2}(a_T)] +$$

$$+ \int_{-\infty}^{+\infty} dx_1 \int_{-\infty}^{+\infty} dy_1 \int_0^{a_T} da \int_{A_T}^{+\infty} dA \frac{1}{\sqrt{2\pi}\sigma_2} \cdot$$

$$\cdot e^{-\frac{[z-f_1(a,A,x_1,y_1)]^2}{2\sigma_2^2}} \frac{1}{2\pi\sigma_1^2} e^{-\frac{x_1^2+y_1^2}{2\sigma_1^2}} .$$

$$\cdot [P_{1A}p_{A_1}(A)+P_{1A}p_{A_2}(A)F_{A1}(A_T) +$$

$$+P_{2A}p_{A_2}(A)+P_{2A}p_{A_1}(A)F_{A2}(A_T)] \cdot$$

$$\cdot [P_{1a}p_{a_2}(a)F_{a1}(a_T)+P_{2a}p_{a_1}(a)F_{a2}(a_T)] +$$

$$+ \int_{-\infty}^{+\infty} dx_1 \int_{-\infty}^{+\infty} dy_1 \int_{a_T}^{+\infty} da \int_{A_T}^{+\infty} dA \frac{1}{\sqrt{2\pi}\sigma_2} \cdot$$

$$\cdot e^{-\frac{[z-f_1(a,A,x_1,y_1)]^2}{2\sigma_2^2}} \frac{1}{2\pi\sigma_1^2} e^{-\frac{x_1^2+y_1^2}{2\sigma_1^2}} .$$

$$\cdot [P_{1A}p_{A_1}(A)+P_{1A}p_{A_2}(A)F_{A1}(A_T) +$$

$$+P_{2A}p_{A_2}(A)+P_{2A}p_{A_1}(A)F_{A2}(A_T)] \cdot$$

$$\cdot [P_{1a}p_{a_1}(a)+P_{1a}p_{a_2}(a)F_{a1}(a_T) +$$

$$+P_{2a}p_{a_2}(a)+P_{2a}p_{a_1}(a)F_{a2}(a_T)] \quad (10)$$

The signals A_1, A_2, a_1, a_2 have Rayleigh distributions:

$$p_{A_1}(A_1) = \frac{A_1}{\sigma_3^2} e^{-\frac{A_1^2}{2\sigma_3^2}} \quad (11)$$

$$p_{A_2}(A_2) = \frac{A_2}{\sigma_4^2} e^{-\frac{A_2^2}{2\sigma_4^2}} \quad (12)$$

$$p_{a_1}(a_1) = \frac{a_1}{\sigma_5^2} e^{-\frac{a_1^2}{2\sigma_5^2}} \quad (13)$$

$$p_{a_2}(a_2) = \frac{a_2}{\sigma_6^2} e^{-\frac{a_2^2}{2\sigma_6^2}} \quad (14)$$

with cumulative probability densities (CDFs) given by:

$$F_{A_1}(A_1) = 1 - e^{-\frac{A_1^2}{2\sigma_3^2}} \quad (15)$$

$$F_{A_2}(A_2) = 1 - e^{-\frac{A_2^2}{2\sigma_4^2}} \quad (16)$$

$$F_{a_1}\left(a_1\right)=1-e^{-\frac{a_1^2}{2\sigma_5^2}} \quad (17)$$

$$F_{a_2}\left(a_2\right)=1-e^{-\frac{a_2^2}{2\sigma_6^2}} \quad (18)$$

After putting the expressions (11) to (18) into (10) we obtain the probability density of the output signal *z*. By means of the output signal pdf other system performances could be calculated.

3. NUMERICAL RESULTS

The probability density function curves (PDFs), *p(z)*, of the signal *z* at the Earth receiver station output, for different parameters are given in Figures 3 to 6.

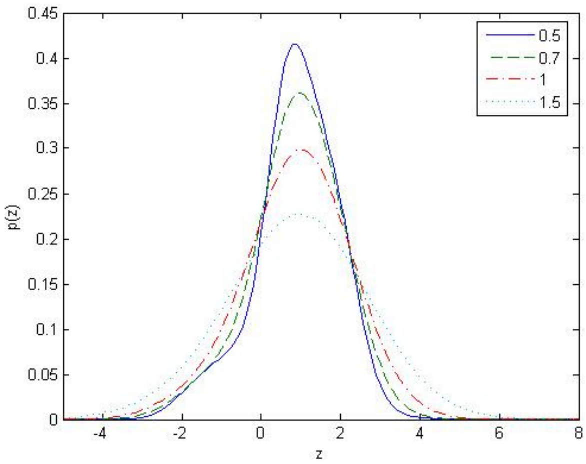

Fig.3. The probability density function *p(z)*, for different parameters: $\sigma_1=1$, $\sigma_2=0.5, 0.7, 1, 1.5$, $\sigma_3=1$, $\sigma_4=1$, $\sigma_5=1$, $\sigma_6=1$, $a_t=1$, $A_t=1$

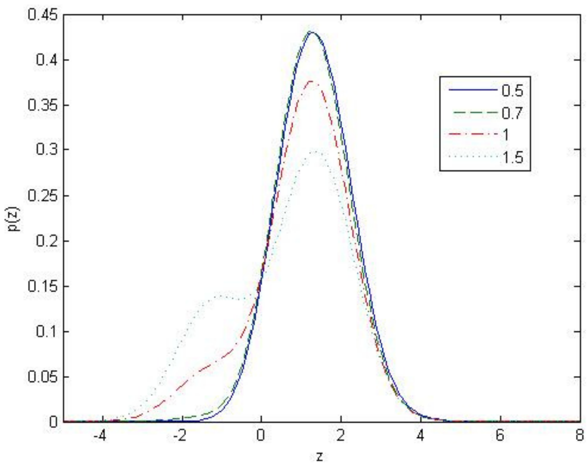

Fig.4. The probability density function *p(z)*, for different parameters: $\sigma_1=0.5, 0.7, 1, 1.5$, $\sigma_2=0.7$, $\sigma_3=1$, $\sigma_4=1$, $\sigma_5=1$, $\sigma_6=1$, $a_t=1$, $A_t=1$

4. SYSTEM PERFORMANCES

The closed form expression for the probability density function (PDF) of the output signal after diversity combining can be used to study the moments, the amount of fading, the outage probability and the average bit error rate of proposed system.

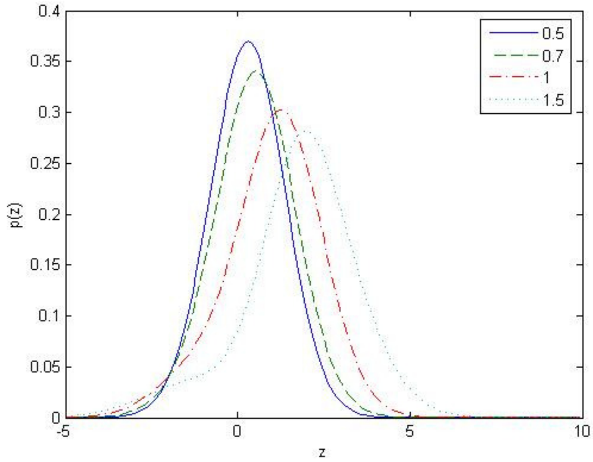

Fig.5. The probability density function *p(z)*, for different parameters: $\sigma_1=1$, $\sigma_2=0.7$, $\sigma_3=0.5, 0.7, 1, 1.5$, $\sigma_4=0.5, 0.7, 1, 1.5$, $\sigma_5=0.5, 0.7, 1, 1.5$, $\sigma_6= 0.5, 0.7, 1, 1.5$, $a_t=1$, $A_t=1$

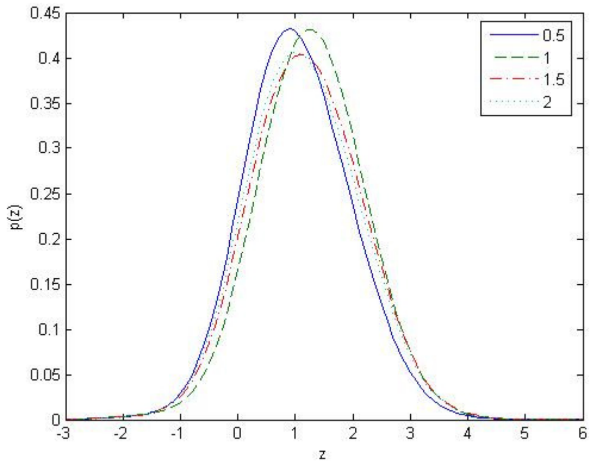

Fig.6. The probability density function *p(z)*, for different parameters: $\sigma_1=0.7$, $\sigma_2=0.7$, $\sigma_3=1$, $\sigma_4=1$, $\sigma_5=1$, $\sigma_6=1$, $a_t=0.5, 1, 1.5, 2$, $A_t= 0.5, 1, 1.5, 2$

The outage probability P_{out} is standard performance criterion of communication systems operating over fading channels. This performance measure is commonly used to control the noise or cochannel interference level, helping the designers of wireless communications system's to meet the quality-of-service (QoS) and grade of service (GoS) demands.

In the interference-limited environment, the outage probability P_{out} is defined as the probability which combined SIR falls below a given outage threshold γ_T, also known as a protection ratio. Protection ratio depends on modulation technique and expected QoS.

The outage probability $P_{out}(r_{th})$ is defined as:

$$P_{out}(r_{th}) = \int_0^{r_{th}} p_r(r)dr\,.$$

For binary phase shift keying (BPSK) modulation scheme, the bit error rate (BER) is given by

$$P_b(e)=\int_0^{\infty} P_b(e/\gamma)p_\gamma(\gamma)d\gamma$$

where $P_b(e/\gamma)$ is conditional BER and $p(\gamma)$ is the PDF of the instantaneous SNR. $P_b(e/\gamma)$ can be expressed in terms of the Gaussian Q-function as

$$P_b(e/\gamma)=Q\left(\sqrt{2g\gamma}\right);$$

Q is the one-dimensional Gaussian Q-function

$$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}dt\,.$$

Gaussian Q-function can be defined using alternative form as

$$Q(x)=\frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{x^2}{2\sin^2\phi}\right)d\phi\,.$$

For coherent BPSK modulation parameter g is determined as $g=1$ and $P_b(e/\gamma)$ is given by

$$P_b(e/\gamma)=Q\left(\sqrt{2\gamma}\right).$$

5. CONCLUSION

In this paper the satellite communication system consisting of the earth transmitting station and the satellite transponder is considered. SSC diversity technics are used on receiving satellite and receiving Earth stations to reduce fading influence to the system performances. The presence of Rayleigh fading on receiving satellite and receiving earth stations is observed. The probability density functions (PDFs) of the signal at the Earth receiver station output are represented for different parameter values.

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