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SOME RESULTS OF TESTING THE DEVELOPED MATLAB® SIMULINK BASED FREQUENCY RESPONSE ANALYZER
NEKI REZULTATI TESTIRANJA RAZVIJENOG FREKVENCIJSKOG ANALIZATORA ZASNOVANOG NA MATLAB® SIMULINKU

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Abstract – *The application of the correlation technique in the system frequency response analysis is presented in this paper. The frequency response analyzer is implemented in MATLAB® Simulink environment and has been tested under different working conditions. Two examples are selected to stress the good features of the analyzer. An electromechanical system has served to show that the remarkable frequency analysis results can be obtained even under conditions of significant additive effects of signal noise on the system output. In another example, by using the developed analyzer the frequency characteristics of a system with non-rational transfer function are investigated.*

Sadržaj – *U ovom radu je prezentovana primena korelacione tehnike pri analizi frekvencijskog odziva sistema. Frekvencijski analizator je implementiran u MATLAB® Simulink okruženju i testiran pod različitim uslovima rada. Odabrana su dva primera kako bi se istakle dobre osobine analizatora. Tako je na primeru jednog elektromehaničkog objekta pokazano da se zavidni rezultati frekvencijske analize dobijaju i u uslovima dejstva značajnog aditivnog signala šuma na izlazu sistema. U drugom primeru, pomoću razvijenog analizatora istražene su frekvencijske karakteristike sistema čije se ponašanje može okarakterisati neracionalnom funkcijom prenosa.*

1. INTRODUCTION

A very practical and important approach to the analysis and design of a control system is the frequency response method. One of the advantages of this approach is the ability to focus the efforts on the interesting frequency ranges. Although the experimental determination of the system frequency responses can be easily accomplished, the main disadvantage is that many industrial processes do not admit sinusoidal inputs in normal operation. The frequency response analysis offers some useful insights into stability and other characteristics of the control system. Frequency response allows us to understand the system behavior in the presence of more complex inputs. Also, frequency response analysis is a very well established system identification approach [1]. Namely, the measurement of the frequency response functions is an intermediate step in the identification process of the nonparametric models of the considered systems.

Notice that the standard Simulink block library does not provide any Frequency Response Analyzer (FRA) block. Hence, in the literature [2], [3] well-known correlation method was implemented in MATLAB® Simulink environment and some results are given in [4], [5], demonstrating the merits of the applied approach.

The paper is organized as follows. First, a brief review of the frequency response analysis by using the correlation method is given in Section 2. Then, in Section 3 some good characteristics of the developed analyzer are visualized by two interesting examples. Finally, in Section 4 some concluding remarks are given.

2. FREQUENCY RESPONSE ANALYZER – THE CORRELATION APPROACH

The frequency response analysis is a simple method for obtaining the detailed information about the considered linear system. The steady-state response $y(t)$ of a stable system to an input

$$u(t) = a\sin(\omega t) \tag{1}$$

is given by

$$y(t) = b\sin(\omega t + \varphi) + n(t), \tag{2}$$

where t is the time, ω is the angular frequency, and $n(t)$ is a measurement noise signal. If we let $G(s)$ be the system transfer function, then the amplitude and phase of the system frequency response can be computed as

$$\left|G(j\omega)\right| = \frac{b}{a}, \qquad \angle G(j\omega) = \varphi(\omega) . \tag{3}$$

Thus, several sinusoidal test signals at different frequencies should be used to measure points on the system frequency response (NYQUIST diagram or BODE plots). The not-so-simple problems of significant noise and/or process nonlinearity that usually occur in engineering systems can be handled with a setup given in Fig. 1 [2]. Moreover, both problems of noise corruption and non-linear distortion are overcome in the measurement scheme in Fig. 1. First of all the measured system output is multiplied with a sine and cosine signal of the frequency of the system input ω. The products are then integrated for a specified measurement time T_m. As the averaging time increases, the contribution of all unwanted frequency components in $y(t)$ goes to zero, and the integrator outputs ($y_s(T_m), y_c(T_m)$) become constant values that depend only on the gain and phase of the considered system transfer function at the test frequency. This experiment should be repeated for a number of frequencies in a certain frequency band.

Therefore, the gain and phase of the system frequency response can be calculated using the following equations:

$$|G(j\omega)| = \frac{2}{aT_m}\sqrt{y_s^2(T_m) + y_c^2(T_m)} \ ,$$

and (4)

$$\angle G(j\omega) = \text{arctg}\frac{y_s(T_m)}{y_s(T_m)} \ .$$

Note that it is recommended to use the four quadrant inverse tangent function, as well as the unwrap algorithm in order to keep the phase continuous over the π - borders.

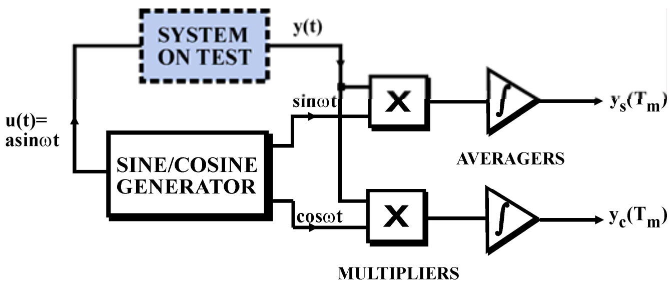


Fig.1. *Block-diagram of the correlation frequency response analyzer*

The above correlation method is based on a few ***assumptions*** that should be done.

♦ Taking into account some *a priori* information about the considered process, it is possible to choose the proper frequency interval for testing.

♦ In determining the amplitude of the input signal $u(t)$, the linearity of the system should be preserved.

♦ The desired accuracy and low noise sensitivity can be achieved by selecting a sufficient long measurement time T_m that should be an integer multiple of the test frequency period T_o, i.e.

$$T_m = \frac{kT_o}{2}\ , \quad k = 1,2,3,\ldots \ . \tag{5}$$

Under these conditions, the average of the integrated noise is zero, which implies that the desired accuracy can be achieved even in case of low signal-to-noise ratio (SNR) [2].

Since the standard Simulink block library does not provide any Frequency Response Analyzer (FRA) block, the above described correlation method is implemented in MATLAB®Simulink, performing all necessary calculations automatically for each value specified in the angular frequency vector [4].

To demonstrate the merits of the correlation method the developed MATLAB®Simulink based frequency response analyzer (FRA) can be used in case of the linear time-invariant system with known transfer function, and compare the obtained characteristics with diagrams provided by using MATLAB® in-built function bode or nyquist. In this paper, the described method will be illustrated with two examples.

3. PRACTICAL EXAMPLES

3.1 *Mass-Spring-Damper System*

Fig. 2 vizualizes one of the variety of configurations to be obtaind with the ECP Model 210 Rectilinear Plant by using springs of varying stiffness [6]. A single drive motor provides actuation to the system via the first mass, and position measurements $x_i(t)$, $i = 1,2$ are taken by quadrature encoders.

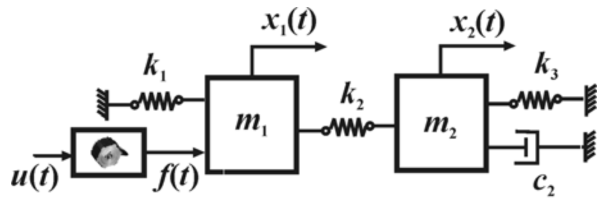


Fig.2. *The scheme of the electro-mechanical plant*

The equations for the considered mass-spring system may be found using Newton's laws to write force balance equations in matrix notation as

$$\mathbf{m}\ddot{\mathbf{x}}(t) + \mathbf{c}\dot{\mathbf{x}}(t) + \mathbf{k}\mathbf{x}(t) = \mathbf{F}(t)\ ,$$

where (6)

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},\ \mathbf{m} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix},\ \mathbf{c} = \begin{bmatrix} 0 \\ c_2 \end{bmatrix},$$

$$\mathbf{k} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}, \text{ and } \ \mathbf{F}(t) = \begin{bmatrix} f(t) \\ 0 \end{bmatrix}.$$

Using the values of the parameters that can be found in the literature [5],[6], the above motion equation results in transfer function vector as follows

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{X_1(s)}{F(s)} \\ \dfrac{X_2(s)}{F(s)} \end{bmatrix} = \begin{bmatrix} \dfrac{0.5882s^2 + 4.412s + 612.7}{\Delta(s)} \\ \dfrac{392.2}{\Delta(s)} \end{bmatrix},$$

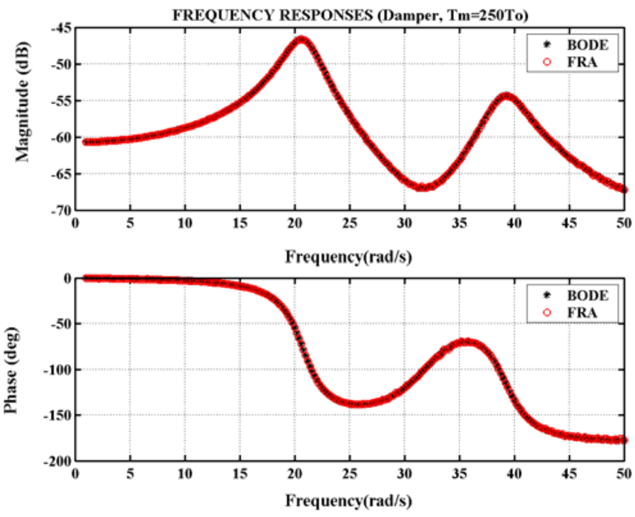
where (7)

$$\Delta(s) = s^4 + 7.5s^3 + 1983s^2 + 7059s + 666700\ .$$

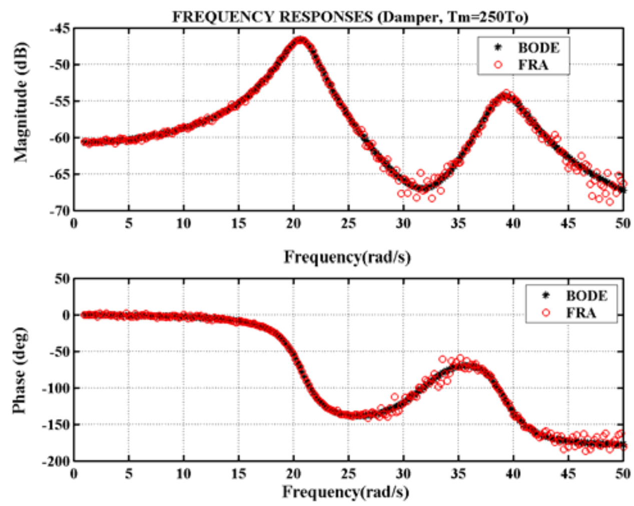
Thus, the plant models are fourth order with two lightly damped poles and either two or no zeros. Below, we discuss only the first plant model with transfer function $G_1(s) = X_1(s)/F(s)$.

Suppose that the output signal $x_1(t)$ has been corrupted by some additive noise. In this subsection we will demonstrate that, under ***assumptions*** given in the previous section, the developed frequency response analizer (FRA) will be robust respect to the significant measurement errors.

Namely, suppose that the analzyer is implemented by using a corrupted measurement $y(t)$, instead of the true system output $x_1(t)$. The response data, estimated by using FRA, can be visualized in two different ways: via the NYQUIST diagram, or via the BODE plots. Both views are shown in Figs. 3 and 4, and as can be seen, the estimated responses approach the frequency characteristics obtained with the mathematical input/output model (7). Moreover, there is a complete matching of the responses when the signal-to-noise ratio (SNR) is approximately equal to 1 (Fig. 3a). Minor deviations can be seen in Figs. 3b and 4 for ten times smaller signal-to-noise ratio.



(a)



(b)

Fig.3. *Frequency responses of the subsystem given by transfer function* $G_1(s)$ *for different SNR values:* (a) $SNR \approx 1$; (b) $SNR \approx 0.1$.

3.2 *First- order hold device*

It is well-known that the output of the first-order hold (FOH) can be described as

$$x_{h1}(t) = x(kT) + \frac{x(kT) - x\left[(k-1)T\right]}{T}(t - kT) \qquad (8)$$

$kT \le t < (k+1)T$, $k = 1, 2, \ldots$, T – is sampling period, and $x(t)$ – is input signal.

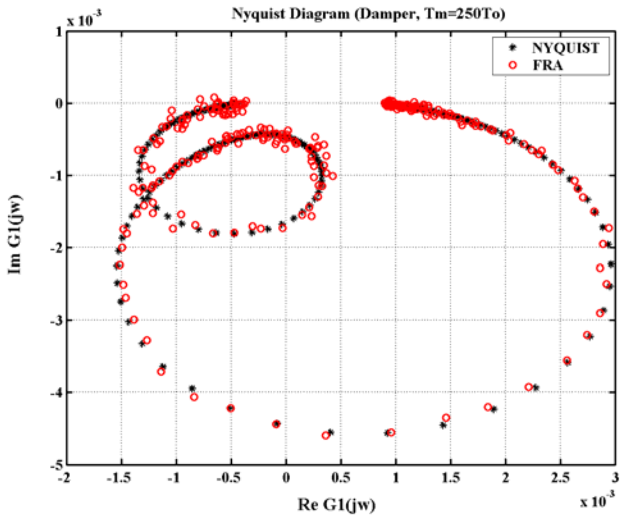


Fig.4. *The NYQUIST plots for mass-spring damper system example and* $SNR \approx 0.1$.

Consider the control structure given in Fig. 5. In order to show that this circuit implements a first-order hold [7], let we define the auxiliary transfer function to be

$$G(s) = \frac{1}{Ts} G_{h0}(s), \qquad (9)$$

where $G_{h0}(s) = \dfrac{1 - e^{-sT}}{s}$ is the zero-order hold transfer function. Now, we can write relations among Laplace transform as

$$C(s) = [G_{h0}(s) + G(s)]E^*(s), \qquad (10)$$

$$E(s) = X(s) - G(s)E^*(s), \qquad (11)$$

and after sampling $$E^*(s) = \frac{X^*(s)}{1 + G^*(s)}. \qquad (12)$$

We can readily calculate that

$$G^*(s) = \frac{e^{-Ts}}{1 - e^{-Ts}}, \qquad (13)$$

and if we substitute (12) and (13) in (10) we obtain

$$C(s) = G_{h0}(s)\left(1 + \frac{1}{Ts}\right)\left(1 - e^{-Ts}\right)X^*(s) = G_{h1}(s)X^*(s). \qquad (14)$$

Then the associated transfer function of the FOH is

$$G_{h1}(s) = \frac{Ts + 1}{T} G_{h0}^2(s). \qquad (15)$$

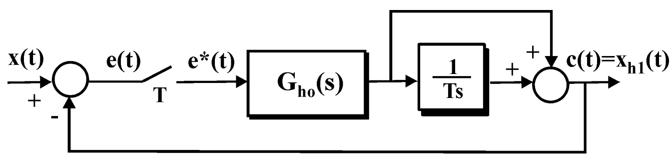


Fig.5. *Block-diagram of the first-order hold device*

Since the transfer function (15) is not a rational function, it is suitable to find a finite-dimensional rational transfer function $G_{9,10}(s)$ by using PADÉ approximation of order $(9,10)$. The information used for this example was taken from the paper [4].

Fig. 6 visualizes the sinusoidal input $x(t)$, the output of the sampler/first-order hold $x_{h1}(t)$, as well as the response of the function $G_{9,10}(s)$ corrected by T according to sampling theory. A relative low sampling frequency of 10 samples per cycle is adopted.

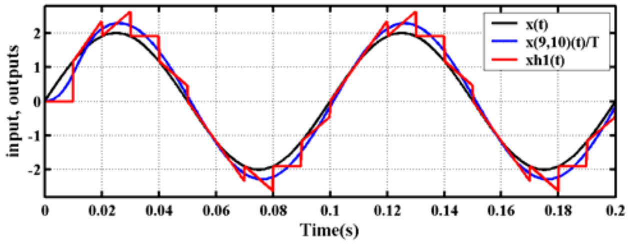


Fig.6. *Responses of FOH to sine wave input*

Some results of frequency response analysis are presented in the form of the BODE and NYQUIST diagrams, which are given for sampling period $T = 0.01\,\mathrm{s}$ in Figs. 7 and 8, respectively. The curves denoted by 'circle' symbol represent the frequency responses of FOH device determined by the considered correlation technique. The curves denoted by 'star' symbol are the BODE (NYQUIST) responses of the rational approximation $G_{9,10}(s)$ of the transfer function $G_{h1}(s)$ given by (15). It can be concluded that the correlation technique provides a quite fine estimation of the frequency responses, with good fits to the amplitude and phase of $G_{9,10}(s)$. However, at higher frequencies the responses (the phase responses, particularly) are distorted, which can be mainly attributed to approximation errors.

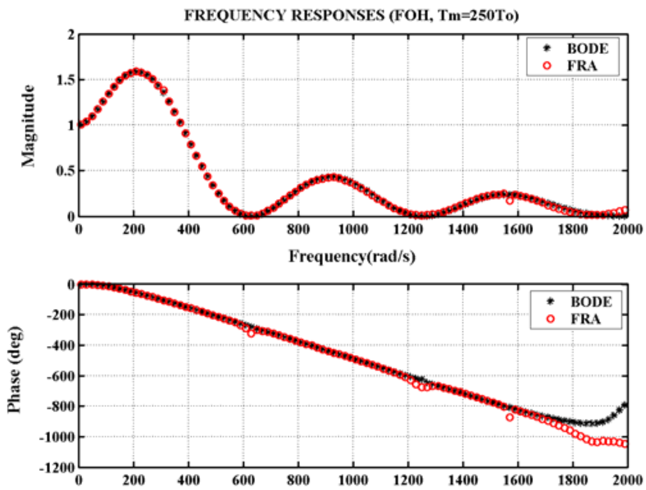


Fig.7. *Frequency responses of first-order hold device for sampling time of* $T = 0.01\,\mathrm{s}$

4. CONCLUSION

The paper presents a frequency response analyzer based on the correlation technique and implemented in MATLAB®Simulink environment. To verify that the developed analyzer works properly, it was tested on two examples. The first example is an electromechanical system that works even under the condition of low signal-to-noise ration, and the second one is a circuit with non-rational transfer function. Based on the presented results it is possible to conclude that the developed analyzer provides a quite fine estimation of the system frequency responses.

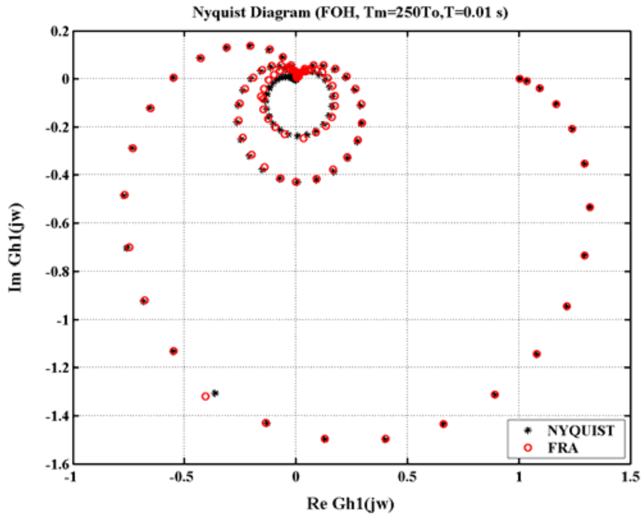


Fig.8. *The* NYQUIST *plots for first-order hold device example*

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