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Level crossing rate of SC receiver output signal operating over Rician multipath fading channel

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Abstract - **Wireless communication system with SC receiver operating over multipath fading channel is considered. Received signal experiences Rician multipath fading. SC diversity receiver is used to reduce Rician fading effects on level crossing rate. Closed form expression for average level crossing rate of SC receiver output signal is calculated. This result is used for evaluation of average fade duration of the proposed system. Numerical expressions are given graphically to show Rician fading effects on average level crossing rate of SC receiver output signal.**

Key words-level crossing rate; Rician fading; Average fade duration;

I. INTRODUCTION

Short term fading degrades system performance and limits channel capacity. Received signal experiences multipath fading resulting in signal envelope variation. There are many statistical modes which can be used to describe signal envelope variation in fading environments depending on propagation environment and propagation scenario [7, 11]. The most frequent distributions which can be used to analyse signal envelope variations are Rayleigh, Rician, Nakagami-m and α-μ [4, 5]. Rayleigh distribution can be used to module signal envelope variation in linear, non-line-of-site multipath fading environments. In linear line-of-site multipath fading channel, small scale signal envelope variations can be described using Rician distribution. This has four parameters. Rician factor k is defined as ratio of dominant component power and scattering components power. By setting parameter k=0, Rician distribution reduces to Rayleigh distribution. System performance is better as Rician parameter increases. Rician fading becomes more severe for lower values of Rician factor. Nakagami-m fading can be used to describe small scale signal envelope variation in linear non-line-of-site multipath fading environments. Squared Nakagami-m random variable can be written as sum of 2m independent squared Gaussian random variables with zero means and the same variances. Fading severity increases as parameter m decreases. By setting parameter m=1 Rayleigh distribution can be derived from Nakagami-m distribution. System performance is better for higher values of parameter m. α-μ distribution can be obtained from Rayleigh, Weibull and Nakagami-m distribution. By setting for α=2 the α-μ distribution reduces to Nakagami-m distribution and by setting for μ=1 Weibull distribution can be obtained from α-μ distribution.

The α-μ distribution approximates Rayleigh distribution by setting for α=2 and μ=1. There are several combining techniques which can be used to reduce multipath fading effects on system performance depending on complexity restriction put on wireless communication system and required performance on system. The most frequently combining techniques are maximum ratio combining (MRC), equal gain combining (EGC) and selection combining (SC). The MRC enables the best outage probability but has the highest implementation complexity. EGC is intermediate solution. The SC has least implementation complexity. The SC receiver selects the branch with the strongest signal. Outage probability and bit error rate are the first order performance measure of wireless communication system. The second order performance measure of communication system are average level crossing rate and average fade duration. Average level crossing rate can be calculated as average value of the first derivative of random variable. Average fade duration can be calculated as ratio of outage probability and average level crossing rate. There are more works which consider second order statistics of wireless communication system in open technical literature [8, 9]. In paper [12] wireless communication system with dual branches, SIR based SC receiver operating over Rician fading channels in the presence of co-channel interference subjected to Rayleigh multipath fading. Closed form expression for average level crossing rate and average fade duration are calculated. In paper [1] macrodiversity with two microdiversity systems operating over shadowed multipath fading channels is considered. Received signal is subjected simultaneously to Nakagami-m multipath fading and Gamma shadowing fading. Level crossing rate and average fade duration of proposed wireless system are calculated. In this paper wireless communication system with SC receiver operating over multipath fading environment is considered. Received signal experiences Rician multipath fading resulting in average fade duration degradation. The SC receiver is used to reduce multipath Rician fading effect on average fade duration of wireless communication system. In this work, average level crossing rate of SC receiver output signal envelope and average fade duration of considered wireless system are evaluated.

II. LEVEL CROSSING RATE OF RICIAN RANDOM VARIABLE

Rician distribution can be used to describe small scale signal envelope variation in linear, line-of-site multipath fading environments. This distribution has two parameters. The parameter k is Rician factor. Rician factor is defined as ratio of dominant component power and scattering components power. Rician factor is higher for grater values of dominant component power. As Rician factor k decreases, fading severity increases. Sistem performance is better for higher values of Rician factor k. Parameter r is average square values of Rician random variables. Parameter r is power of Rician random variable. By setting for Rician factor k=0, Rician random variable reduces for Rayleigh distribution. Rician random variable can be written as sum of two squared Gaussian random variables.

$$x^2 = x_1^2 + x_2^2 \tag{1}$$

Where x_1 and x_2 are Gaussian random variables with variances . The first derivative of Rician random variable is:

$$\dot{x} = \frac{1}{x}(x_1\dot{x}_1 + x_2\dot{x}_2) \tag{2}$$

The first derivative of the Gaussian random variable is Gaussian random variable. Linear transformation of Gaussian random variable is Gaussian random variable. Therefore, the first derivative of Rician random variable follows Gaussian distribution. The mean of $\dot{x}$ is:

$$\bar{\dot{x}}=\frac{1}{x}(x_1\bar{\dot{x}}_1+x_2\bar{\dot{x}}_2) \tag{3}$$

since

$$\bar{\dot{x}}_1=\bar{\dot{x}}_2=0$$

Variance of the first derivative of Rician random variable is:

$$\sigma_{\dot{x}}^2=\frac{1}{x^2}(x_1^2\sigma_{\dot{x}_1}^2+x_2^2\sigma_{\dot{x}_2}^2) \tag{4}$$

where

$$\sigma_{\dot{x}_1}^2=\sigma_{\dot{x}_2}^2=\pi^2 f_m^2\sigma^2$$

where f_m is maximal Doppler frequency.

After substituting, the expression for variance of $\dot{x}$, becomes:

$$\sigma_{\dot{x}}^2=\frac{1}{x^2}2\pi^2 f_m^2\sigma^2(x_1^2+x_2^2)= 2\pi^2 f_m^2\sigma^2=\Omega\pi^2 f_m^2 \tag{5}$$

The probability density function of the first derivative of Rician random variable is:

$$p_{\dot{x}}(\dot{x})=\frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}}e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}} \tag{6}$$

The joint probability density function of Rician random variable and the first derivative of Rician random variable is:

$$p_{x\dot{x}}(x\dot{x})= p_{\dot{x}}(\dot{x}/x)p_x(x)= p_{\dot{x}}(\dot{x})\, p_x(x) \tag{7}$$

where

$$p_x(x)=\frac{2(1+k)x}{e^k r}e^{-\frac{(1+k)x^2}{r}}I_0(2\sqrt{\frac{k(1+k)}{r}}\,x),$$
$$k=\frac{A^3}{2\sigma^2}=\frac{A^3}{\Omega} \tag{8}$$

Level crossing rate of Rician random variable is:

$$N_x=\int_0^\infty d\dot{x}\dot{x}p_{x\dot{x}}(x\dot{x}) = p_x(x)\int_0^\infty d\dot{x}\dot{x}p_{\dot{x}}(\dot{x}) = p_x(x)\int_0^\infty d\dot{x}\dot{x}\frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}}e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}} = p_x(x)\frac{1}{\sqrt{2\pi}}\sigma_{\dot{x}} = \frac{2(1+k)x}{e^k r}e^{-\frac{(1+k)}{r}x^2}I_0(2\sqrt{\frac{k(1+k)}{r}}x)\frac{1}{\sqrt{2\pi}}\Box f_m r^{1/2} = \sqrt{2\pi}f_m\frac{(1+k)x}{e^k r^{1/2}}e^{-\frac{(1+k)}{r}x^2}I_0(2\sqrt{\frac{k(1+k)}{r}}\,x) \tag{9}$$

The cumulative distribution function of Rician random variable:

$$F_x(x)=\int_0^x p_x(t)\,dt= \int_0^x dt\frac{2(1+k)}{e^k r}te^{-\frac{(1+k)}{r}t^2}\sum_{i=0}^\infty(\sqrt{\frac{k(1+k)}{r}})^{2i}\frac{1}{i!}t^{2i}= \frac{2(1+k)}{e^k r}\sum_{i=0}^\infty(\sqrt{\frac{k(1+k)}{r}})^{2i}\frac{1}{i!^2}\int_0^x dt\,t^{2i+1}e^{-\frac{(1+k)}{r}t^2}= \frac{2(1+k)}{e^k r}\sum_{i=0}^\infty(\sqrt{\frac{k(1+k)}{r}})^{2i}\frac{1}{i!^2}\frac{1}{2}(\frac{r}{1+k})^{i+1}\Gamma(i+1,\frac{1+k}{r}x^2) \tag{10}$$

The expression of average level crossing rate and cumulative distribution function of Rician random variable can be used for evaluation of outage probability and average fade duration of wireless communication system with SC receiver operating over Rician multipath environment.

III. LEVEL CROSSING RATE OF SC RECEIVER OUTPUT SIGNAL ENVELOPE

Wireless communication system with dual branch SC receiver operating over Rician multipath fading channel is considered. Signal envelope at inputs of SC receiver are denoted with x_1 and x_2. Joint probability density function of SC receiver output signal envelope and the first derivative of SC receiver output signal envelope is:

$$p_{x\dot{x}}(x\dot{x})=p_{x_1\dot{x}_1}(x\dot{x})F_{x_1}(x) + p_{x_2\dot{x}_2}(x\dot{x})F_{x_2}(x) \tag{11}$$

Where $F_{x_1}(x)$ and $F_{x_2}(x)$ are cumulative distribution functions of random variables x_1 and x_2 respectivly and $p_{x_1\dot{x}_1}(x\dot{x})$ and $p_{x_2\dot{x}_2}(x\dot{x})$ joint probability density functions of x_1 and $\dot{x}_1$ and joint probability density functions of x_2 and $\dot{x}_2$ respectivly. For identical Rician fading, previous exoression can be written in the form:

$$p_{x\dot{x}}(x\dot{x})=2p_{x_1\dot{x_1}}(x\dot{x})\,F_{x_2}(x) \tag{12}$$

Level crossing rate of SC receiver output signal is [6]:

$$N_x=\int_0^\infty d\,\dot{x}\dot{x}p_{x\dot{x}}(x\dot{x})=\int_0^\infty d\,\dot{x}\dot{x}2p_{x_1\dot{x_1}}(x\dot{x})\,F_{x_2}(x)=$$
$$2F_{x_2}(x)\int_0^\infty d\,\dot{x}\dot{x}2p_{x_1\dot{x_1}}(x\dot{x})=2F_{x_2}(x)\,N_{x_1}=$$
$$=\frac{2(1+k)}{e^k r}\sum_{i=0}^{\infty}\left(\sqrt{\frac{k(1+k)}{r}}\right)^{2i}\frac{1}{(i!)^2}\left(\frac{r}{1+k}\right)^{i+1}$$
$$\Gamma\left(i+1,\frac{1+k}{r}x^2\right)$$
$$f_m\sqrt{2\pi}\frac{(1+k)x}{e^k r^{1/2}}e^{-\frac{(1+k)}{r}x^2}I_0\left(2\sqrt{\frac{k(1+k)}{r}}\,x\right) \tag{13}$$

Outage probability of wireless system with SC receiver is:

$$F_x(x)=F_{x_1}(x)\,F_{x_2}(x)=(F_{x_1}(x))^2 \tag{14}$$

Average fade duration of wireless system is:

$$AFD=\frac{P_0}{N_x}=\frac{F_{x_1}(x)}{2N_x}=$$
$$\frac{\frac{1+k}{e^k r}\sum_{i=0}^{\infty}\left(\sqrt{\frac{k(1+k)}{r}}\right)^{2i}\frac{1}{(i!)^2}\left(\frac{r}{1+k}\right)^{i+1}\Gamma\left(i+1,\frac{1+k}{r}x^2\right)}{f_m\sqrt{2\pi}\frac{(1+k)x}{e^k r^{1/2}}e^{-\frac{(1+k)}{r}x^2}I_0\left(2\sqrt{\frac{k(1+k)}{r}}\,x\right)} \tag{15}$$

In Fig. 1, average level crossing rate of SC receiver output signal versus SC receiver output signal envelope is shown for several values of Rician factor k and power of SC receiver output signal envelope. For lower values of SC receiver output signal envelope, average level crossing rate increases as SC receiver signal envelope increases. For higher values of SC receiver output signal envelope, average level crossing rate decreases as SC receiver signal envelope increases. The SC receiver output signal envelope has the greatest influence on average level crossing rate for intermediate values of SC receiver output signal envelope. The average level crossing rate decreases as power of SC receiver output signal envelope increases. System performance is better for lower values of Rician factor. As Rician factor k decreases dominant component power decreases and the average level crossing rate increases.

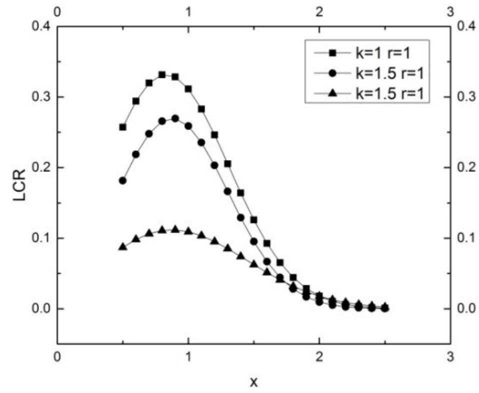

Fig. 1. Level crossing rate versus SC receiver output signal envelope

In Fig. 2, average level crossing rate of SC receiver output signal envelope versus Rician factor k for different values of SC receiver output signal envelope and average signaled values of SC receiver output signal envelope is shown. The average level crossing rate of SC receiver output signal envelope decreases as Rician factor k increases. Rician factor k has greater influence on average level crossing rate for lower values of Rician factor k. Average level crossing rate has higher values for less values of SC receiver output signal envelope and SC receiver output signal envelope power.

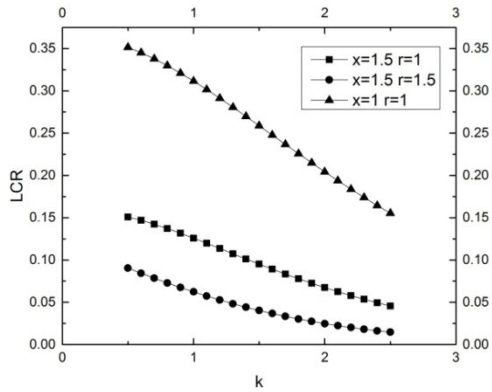

Fig. 2. Level crossing rate versus Rician factor

In Fig. 3, average level crossing rate of SC receiver output signal envelope power for several values of SC receiver output signal envelope and Rician factor k is presented. Average level crossing rate of SC receiver output signal envelope decreases as average signaled values of SC receiver output signal envelope increases. Average signaled values of SC receiver output signal envelope has the greatest influence on average level crossing rate for lower values of average values of SC receiver output signal envelope.

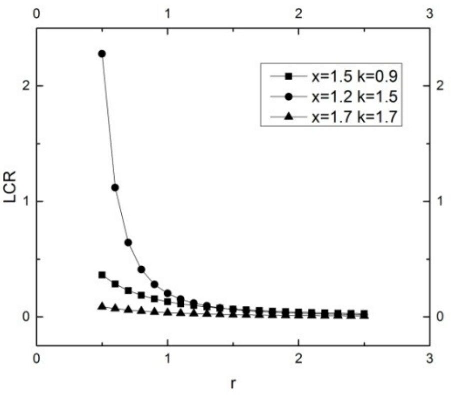

Fig. 3. Level crossing rate versus SC receiver output signal envelope power

IV. CONCLUSION

Wireless communication system with dual SC receiver operating over multipath fading environment is considered. Received signal is subjected to Rician multipath fading resulting in signal envelope variation. The SC receiver is used to reduce multipath Rician fading effect on level crossing rate and average fade duration. Parameter of Rician fading is Rician factor k which is defined as ratio of dominant component power and scattering components power. Rician fading is more severe for lower values of Rician factor k. For lower values of dominant component power and higher values of scattering components power the values of Rician factor k are lower resulting in system performance degradation. Closed form expression for average fade duration and average level crossing rate are calculated. Average level crossing rate is calculated as the average value of the first derivative of SC receiver output signal envelope and average fade duration is determined as ratio of outage probability and average level crossing rate. Outage probability can be calculated as square of cumulative distribution function of Rician random variable. Average level crossing rate can be calculated as double product of average level crossing rate of Rician random variable. Numerical results are presented graphically to show influence of Rician factor k and power of Rician random variable on level crossing rate of SC receiver output signal envelope. For lower values of SC receiver output signal envelope, level crossing rate increases as SC receiver output signal envelope increases and for higher values of SC receiver output signal envelope, level crossing rate decreases as SC receiver output signal envelope increase. Level crossing rate of SC receiver output signal envelope decreases as Rician factor k increases. As power of SC receiver output signal envelope increases average level crossing rate decreases. The influence of power of SC receiver output signal envelope on average level crossing rate is grater for lower values of SC receiver output signal envelope power. Results obtained in this paper can be used in performance analysing and designing wireless communication signals with dual SC receiver operating over multipath Rician fading environment.

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