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# One Solution for the Wide-Angle Beam Propagation Method

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*Abstract*—**Wide-angle beam propagation method is usually based on Padé approximant operators and multi-step algorithms in transverse plane. In this paper the solution is derived which implies a wide-angle multi-step algorithm in the longitudinal direction without using Padé approximant operators. It is shown that this solution enables accurate and efficient modeling of wide-angle beams, without any increase of computational time in comparison to Padé based methods. The presented *z*-multi-step algorithm offers, to the best knowledge of the author, novel wide-angle beam propagation methods in optoelectronics.**

***Keywords—beam propagation method; finite-difference; wide-angle schemes; optoelectronics; numerical simulations***

## I. INTRODUCTION

The finite-difference beam propagation method (FD-BPM) is perhaps the most popular design tool used in integrated optoelectronics and photonics [1]-[6]. Generally speaking, the beam propagation algorithms are defined as approaches for numerical solving of the paraxial approximation of an exact vector Helmholtz equation (also known as the Fresnel equation).

Since the first formulation of the FD-BPM in 1990 [7], the original FD-BPM has undergone significant improvements. The advantages of the FD-BPM include its simple and flexible numerical implementation, and its fast convergence. On the other hand, the main feature of the original FD-BPM, the paraxial approximation, at the same time presents serious drawback of the FD-BPM algorithm, because of the limitation of angular range of principal propagation direction. The slowly varying envelope approximation (SVEA) simplifies the FD-BPM algorithm by neglecting the second-order derivative with respect to $z$ in the Helmholtz equation, limiting the field simulations to paraxial beams and weekly guiding cases (waveguide structures with low refractive index contrast ratio between its dielectric core and cladding).

Attempts to overcome this intrinsic limitation of the algorithm have generated several techniques for its correction. On the basis of correction proposed in acoustics [8], [9], a remedy was found by Hadley in the early 1990s [10] when the wide-angle (WA) BPM algorithm using higher-order Padé approximation of the square root operator was introduced. So far, several WA-BPM algorithms have been suggested [11]-[25] allowing improvements of the standard paraxial BPM technique, however offering sometimes complicated mathematical manipulations to complete the efficient WA-FD-BPM propagation algorithm.

By using WA-FD-BPM algorithms the electric or magnetic field can be propagated through tilted and curved waveguide structures and circuits without loss of accuracy and additional local co-ordinate transformations, giving more realistic results of the light-wave propagation. Various implementations and suitable improvements of higher order Padé operator and multi-step approach have been proposed recently [11]-[16]. Apart from Padé operator methods, the BPM algorithms using Hoekstra scheme [17] offer accurate simulations [18], as well as algorithm based on the spectral collocation scheme [19]. Some treatments of WA-FD-BPM without SVEA approach have also been reported [14], [20], [21]. WA-BPM algorithms combined with split-step and ADI methods [22], [23] are proposed as well. The implementation of various improvements of the WA-BPM [24], [25] is perhaps the most attractive area in FD-BPM research.

In this work the scalar Helmoltz equation under the SVEA is formally solved in terms of higher-order field derivatives. Obtained formal solution enables relatively straightforward implementation of the $z$-multi-step operator approach. No Padé operator splitting is present in the derived $z$-multi-step (operator split-step) algorithm. The $z$-multi-step algorithm is easily designed allowing usual FD discretization and simple computer coding. Both guiding and evanescent modes are treated properly with simple modification of the approach. Some numerical examples are added to show efficiency of the proposed $z$-multi-step method.

## II. FORMULATION

In this section the one-way wave equation (Helmholtz equation) under the SVEA approximation is derived in form of the finite sum of field derivatives. In order to highlight only an idea, the wave equation for the magnetic field and for 2D scalar cases is derived only. However, the formulation presented is easily and directly extendable to the electric field case and to 2D semi-vectorial (polarized), 3D scalar, 3D semi-vectorial and, with the inclusion of the improved FD formulas, for 3D full-vectorial cases.

3D scalar Helmholtz equation is given by

$$\frac{\partial^2 \tilde{H}}{\partial x^2} + \frac{\partial^2 \tilde{H}}{\partial z^2} + k_0^2 n^2 \tilde{H} = 0 , \tag{1}$$

for the scalar magnetic field component $\tilde{H} = \tilde{H}(x,z)$ with $n = n(x,z)$ as the refractive index profile, while $k_0$ denotes the free-space wave-number. If the SVEA approximation is assumed the magnetic field can be separated into two parts: a complex field amplitude $H = H(x,z)$ (the slowly varying envelope term) and a propagation factor $\exp(-jkz)$ (the rapidly varying phase term),

$$\tilde{H} = H(x,z) \cdot e^{-jkz} , \tag{2}$$

where $k = k_0 n_{ref}$ , $n_{ref}$ is the reference refractive index, and $j = \sqrt{-1}$ . Substituting (2) into (1) the one-way SVEA wave equation is obtained,

$$\frac{\partial^2 H}{\partial z^2} - j2k \frac{\partial H}{\partial z} = -PH , \tag{3}$$

where the operator $P$ is defined as

$$P = \nabla_T^2 + k_0^2 \left(n^2 - n_{ref}^2\right) = \frac{\partial^2}{\partial x^2} + k_0^2 \left(n^2 - n_{ref}^2\right). \tag{4}$$

By assuming,

$$\frac{\partial H}{\partial z} = f(x,z) = f , \tag{5}$$

we can rewrite (3) as

$$\frac{\partial f}{\partial z} - j2k \cdot f = -PH . \tag{6}$$

Equation (6) is the first order differential equation with (in general) variable coefficients, assumed to be a function of only one variable $z$, an ODE, with the general form (with respect to $z$) given by

$$\frac{\partial f(z)}{\partial z} + \varphi(z) \cdot f(z) = \psi(z) . \tag{7}$$

The solution of (7) is given as

$$f(z) = e^{-a(z)} \left[ \int \psi(z) \cdot e^{a(z)} dz + K \right], \quad a(z) = \int \varphi(z)\, dz , \tag{8}$$

where $K$ is the constant of integration, and

$$\psi(z) = -PH, \quad \varphi(z) = -j2k, \quad a(z) = -j2kz . \tag{9}$$

Equation (8) becomes:

$$f(z) = \frac{\partial H}{\partial z} = e^{+j2kz} \left[ -\int PH \cdot e^{-j2kz} dz + K \right]. \tag{10}$$

The solution of integral in (10) yields a parabolic-type solution of (3). One possibility to solve it is to expand the scalar magnetic field envelope function $H(z)$ in the power series

$$H(z) = a_0 + a_1 (k_0 z) + a_2 (k_0 z)^2 + a_3 (k_0 z)^3 + \cdots , \tag{11}$$

or

$$H(x,z) = \sum_{i=0}^{N} a_i(x) \cdot (k_0 z)^i , \tag{12}$$

where $k_0 z = 2\pi \cdot z / \lambda_0$, with $\lambda_0$ denoting the free-space wave-length. $N$ denotes the order of the series expansion, i.e. the highest order of the field derivative considered in the solution.

### A. *Solution for N = 0*

If we choose $N$=0, i.e. $H(z) = a_0$, (10) becomes

$$\frac{\partial H}{\partial z} = e^{+j2kz} \left[ -\int P a_0 \cdot e^{-j2kz} dz + K \right], \tag{13}$$

leading to the solution

$$\frac{\partial H}{\partial z} = \frac{1}{j2k} P a_0 + K e^{+j2kz} , \tag{14}$$

presenting the equation for the steady state regime of the propagation. The constant of integration $K$ depends of the initial condition of the field distribution at $z=0$. Having in mind that the initial field distribution at $z=0$ in practical situations is always known, we can put that $K=0$. Therefore, we arrived to the zeroth-order BPM approximation of (3),

$$j2k \frac{\partial H}{\partial z} = P a_0, \qquad j2k \frac{\partial H}{\partial z} = PH . \tag{15}$$

### B. *Solution for N = 1*

For $N$=1, i.e. $H(z) = a_0 + a_1(k_0 z)$, we have a linear case, and (10) becomes

$$\frac{\partial H}{\partial z} = e^{j2kz} \left[ \frac{1}{j2k} P a_0 \cdot e^{-j2kz} - P a_1 k_0 \int z \cdot e^{-j2kz} dz \right], \tag{16}$$

and after making use of the partial integration, we obtain

$$j2k \frac{\partial H}{\partial z} = P\left(a_0 + a_1 \cdot k_0 z\right) + P a_1 \cdot \frac{k_0}{j2k} . \tag{17}$$

So, we have finally,

$$j2k\frac{\partial H}{\partial z}=PH+\frac{1}{j2k}P\left(\frac{\partial H}{\partial z}\right). \qquad (18)$$

Equation (18) is the first-order WA-BPM correction of (3), and has the same form as Padé (1,1) WA-BPM solution [1], [13].

*C. Solution for N=2*

For $N$=2, i.e. $H(z)=a_0+a_1(k_0 z)+a_2(k_0 z)^2$, (10) gives

$$\frac{\partial H}{\partial z}=e^{j2kz}\left[-\int P(a_0+a_1\cdot k_0 z+a_2\cdot k_0^2 z^2)\cdot e^{-j2kz}dz\right]. \qquad (19)$$

Integral with $z^2$ which appears in (19),

$$I=Pa_2\cdot k_0^2\int z^2\cdot e^{-j2kz}dz, \qquad (20)$$

can be solved by using the partial integration,

$$I=\int z^2 e^{-j2kz}dz=-\frac{e^{-j2kz}}{j2k}\left[z^2+\frac{2z}{j2k}+\frac{2}{(j2k)^2}\right], \qquad (21)$$

yielding

$$\frac{\partial H}{\partial z}=\frac{Pa_0}{j2k}+\frac{Pa_1}{j2k}\left[k_0 z+\frac{k_0}{j2k}\right]+\frac{Pa_2}{j2k}\left[k_0^2 z^2+\frac{2k_0^2 z}{j2k}+\frac{2k_0^2}{(j2k)^2}\right]. \qquad (22)$$

By rearranging the right hand side of (22), we obtain the second-order WA-BPM correction, or approximation, of (3),

$$j2k\frac{\partial H}{\partial z}=PH+\frac{1}{j2k}P\left(\frac{\partial H}{\partial z}\right)+\frac{1}{(j2k)^2}P\left(\frac{\partial^2 H}{\partial z^2}\right). \qquad (23)$$

*D. Solution for arbitrary N*

Following the same procedure as it is shown for $N$=0,1 and 2, the $N$th-order WA-BPM correction (approximation) formula of the wave equation (3), can be derived as

$$j2k\frac{\partial H}{\partial z}=P\sum_{n=0}^{N}\left[\frac{1}{(j2k)^n}\left(\frac{\partial^n}{\partial z^n}\right)\right]H. \qquad (24)$$

In (24) we assume $\partial^n/\partial z^n\big|_{n=0}\equiv 1$, $P\left(\partial^n/\partial z^n\big|_{n=0}\right)H\equiv PH$. On the basis of the formula for geometric progression [26],

$$a+ar+ar^2+ar^3+\cdots+ar^N\equiv a\frac{1-r^{N+1}}{1-r}, \qquad (25)$$

by introducing a new operator $X$,

$$X=\frac{1}{j2k}\frac{\partial}{\partial z}, \qquad X^n=\frac{1}{(j2k)^n}\frac{\partial^n}{\partial z^n}, \qquad (26)$$

and taking $a=1$, $r=X$ in (25), formula (24) gets the form

$$j2k\frac{\partial H}{\partial z}\equiv P\left[1+X+\cdots+X^N\right]H\equiv P\frac{1-X^{N+1}}{1-X}H. \qquad (27)$$

*E. Multi-step operator solution*

The $N$th-order WA-BPM approximation formulae (24) and (25) give rise for design of, so-called, operator split-step, or multi-step methods. For instance, if we normalize the $z$ co-ordinate with $z_n$, as

$$z=\frac{z_n}{j2k}, \qquad (28)$$

equation (27) becomes

$$(j2k)^2\frac{\partial H}{\partial z_n}=P\left[1+\left(\frac{\partial}{\partial z_n}\right)+\left(\frac{\partial^2}{\partial z_n^2}\right)+\cdots+\left(\frac{\partial^N}{\partial z_n^N}\right)\right]H. \qquad (29)$$

If constant $a$ and a new operator $Q$ are introduced as,

$$a=j2k, \qquad Q=\frac{\partial}{\partial z_n}, \qquad (30)$$

we can obtain

$$\left(a^2 Q\right)H=P\left(1+Q^1+Q^2+\cdots+Q^N\right)H. \qquad (31)$$

By factorizing the polynomial term in (31), we obtain

$$\left(a^2 Q\right)H=P\left[(Q-c_1)\times(Q-c_2)\times\cdots\times(Q-c_N)\right]H, \qquad (32)$$

or finally,

$$\left(a^2 Q\right)H=P\prod_{i=1}^{N}(Q-c_i)H, \qquad (33)$$

where $c_i$, $i=1,\cdots,N$, are complex roots of the operator polynomial expansion in (31).

As an example, for $N$=2, (33) has the form

$$j2k\frac{\partial H}{\partial z}=P\left(c-\frac{1}{j2k}\frac{\partial}{\partial z}\right)\left(c^*-\frac{1}{j2k}\frac{\partial}{\partial z}\right)H, \qquad (34)$$

where $c$* is the complex conjugate of $c$, $c=-\frac{1}{2}+j\frac{\sqrt{3}}{2}$. In (34) we have two operator steps in the multi-step algorithm, each of them is like a single well-known BPM Padé (1,1) algorithm. Note that in (24), (27) and (29) to (34) operators $P$ and $Q$ commute.

Obviously, more steps in (33) is applied, the better accuracy of the field simulation is obtained. This is the same algorithm philosophy as behind Padé operator expansion methods [10], [12]-[15]. However, this novel approach enables operator splitting in the $z$-direction (not in transverse plane, as it was usually performed).

## III. Numerical results

The purpose of this section is to demonstrate effectiveness of novel $z$-directed multi-step WA-FD beam propagation method in the field computation of 2D optoelectronic tilted waveguide for an arbitrary angle of propagation.

A standard benchmark 2D waveguide is selected [27] and the propagation of an eigenmode in a 20 deg. tilted waveguide is simulated by $z$-directed multi-step WA-FD-BPM. 2D structure is a GaAs based symmetrical slab waveguide, with refractive index equal to 3.3704 and 3.2874 in the core and cladding, respectively. The width of the waveguide core is taken to be $w = 2\,\mu\text{m}$. The reference refractive index of $TE_0$ mode is calculated analytically [27], and has a value $n_{ref} = 3.357986936764703$. The operating wave-length is $\lambda_0 = 1.55\,\mu\text{m}$. The cross section of the slab waveguide structure, in $y = const.$ plane, is shown in Fig. 1.

In the field simulations the window size was kept to be $L = 10\,\mu\text{m}$ and FD mesh-size in transverse plane was chosen to be $\Delta x = 0.05\,\mu\text{m}$. The propagation marching step size $\Delta z$ in the principal propagation direction $z$, was changed in the range from $\Delta z = \Delta x = 0.05\,\mu\text{m}$ to $\Delta z = 20 \cdot \Delta x = 1\,\mu\text{m}$.

At the edge of the computational window the transparent boundary conditions were used. At the input of the tilted slab waveguide, Gaussian field distribution (wider than analytically calculated fundamental $TE_0$ mode) was launched.

The magnetic field pattern of $TE_0$ mode at a $\theta = 20$ deg. tilted angle and at propagation distance $L_z = M_z \cdot \Delta z$ is shown in Fig. 2. Very similar results were obtained for all values of tested marching steps $\Delta z$ in aforementioned range. Field plots are shown for $M_z = 6000$ and $\Delta z = 0.1\,\mu\text{m}$, i.e. at the propagation distance $L_z = 6000 \cdot \Delta z = 0.6\,\text{mm}$.

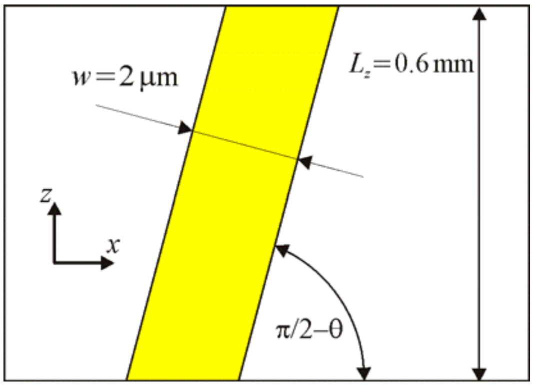


Figure 1. A tilted straight waveguide at $\theta = 20$ degrees.

As can be seen from Fig. 2 and Fig. 3, the propagation of the magnetic field in tilted slab waveguide is better calculated as $N$ is higher. $N$ denotes the number of steps in the $z$-directed multi-step method, but at the same time the highest considered derivative of the magnetic field in the propagation algorithm.

The overall conclusion is that even for $N$=3, accuracy is sufficient, while for $N$=5 and $N$=7 field patterns coincide. It is worth to mention that a slight modification of the WA-FD-BPM algorithm has been performed to enable good computation (with appropriate dumping) of the evanescent field (radiation modes) in the waveguide cladding.

## IV. Conclusion

A novel $z$-directed wide-angle finite-difference beam propagation method (WA-FD-BPM) solution is proposed. Proposed approach and propagation multi-step algorithm seem to have advantages over well-known Padé operator multi-step schemes. The main advantage lays in the fact that the $z$-multi-step algorithm works naturally – in the $z$-direction, which is the principal direction of the propagation. The results of the benchmark test are presented and show that the proposed wide-angle $z$-multi-step algorithm is capable of giving accurate field simulations comparable with well-established wide-angle algorithms based on Padé operator approach.


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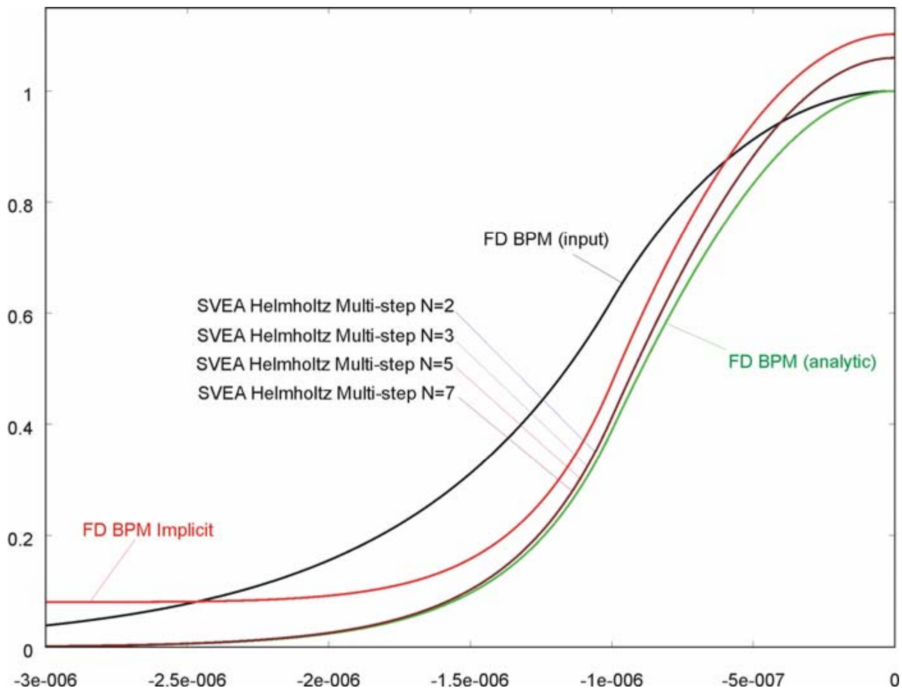


Figure 2. Magnetic field pattern, the left-hand side of a symmetrical tilted slab waveguide.

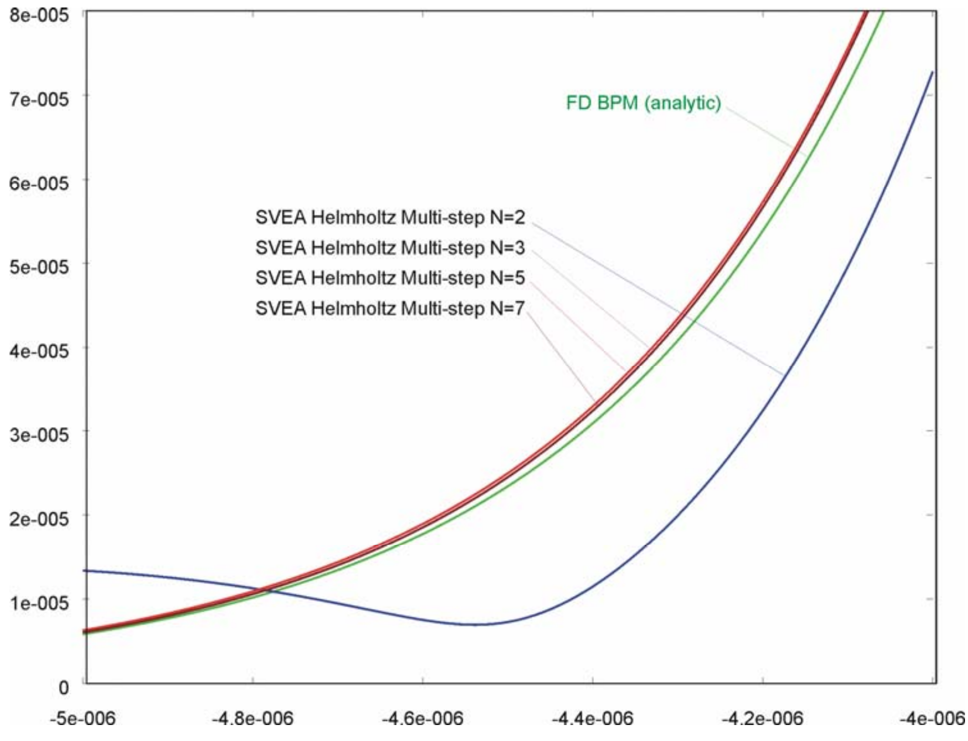


Figure 3. Magnetic field pattern, a zoom in the left-hand side of a symmetrical tilted slab waveguide.

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