PERFORMANSE DIVERZITI SISTEMA U PRISUSTVU VIŠESTRUKIH KO-KANALNIH INTERFERENCIJA I KORELISANOG REJLIJEVOG FEDINGA

PERFORMANCE DIVERSITY SYSTEMS IN THE PRESENCE OF MULTIPLE CO-CHANNEL INTERFERENCE AND CORRELATED RAYLEIGH FADING

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Sadržaj - *U ovom radu je predstavljena analiza performansi prijema višeantenskog prijemnika sa selektivnom diverziti (SC) tehnikom kombinovanja, kada se prenos signala vrši kroz komunikacione kanale sa korelisanim Rejlevim fedingom i u prisustvu višestrukih ko-kanalnih interferencija. Izložena analiza razmatra slučaj uticaja proizvoljnog broja korelisanih ko-kanalnih interferencija po svakom kanalu. Predstavljeni su izrazi u zatvorenom obliku za funkciju gustine verovatnoće (PDF) i kumulativnu funkciju verovatnoće (CDF) odnosa signal/interferencija (SIR) na izlazu iz kombinera na prijemu. Na osnovu ovih statističkih veličina određena je standardna mera performansi sistema bežičnog prenosa i verovatnoća otkaza, čije su vrednosti razmatrane u funkciji raznih parametara sistema.*

Abstract - *This paper provides a performance analysis of selection combining (SC) receiver over the correlated Rayleigh fading channels operating in general interference-limited environment. Analysis of multibranch SC based on signal-to-interference ratio (SIR) is provided for the case when each correlated branch experiences an arbitrary number of multiple correlated co-channel interferers. Closed form expressions with convergence discussion for the SIR's standard first order statistics i.e. probability density (PDF) and cumulative distribution function (CDF), at the SC output are presented. Capitalizing on them, an important performance measure of wireless transmission, the outage probability (OP) was analysed, in order to show the effects of the number of multiple interferers, diversity order, level of correlation and input SIR unbalance on the general transmission characteristics.*

1. INTRODUCTION

Mobile terrestrial and satellite communication systems have been extensively developed in recent years. Their main purpose is to provide the necessary quality of service combined with high capacity [1]. Because of that those systems tend to conserve the available spectrum by reusing allocated frequency channels in areas that are geographically located as close to each other as possible. However, due to frequency reuse, signals from two or more channels operating at the same frequency, but from different locations, interfere. Co-channel interference is defined as the interfering signal that has the same carrier frequency as the useful information signal. The methodology for the analysis of the impact of any kind co-channel interference is given in [2]. The main objective is then to analyse how the interference as a general distortion (in the form of signal-to-interference ratio, SIR) affects well-accepted criteria of performance of wireless systems, such as outage probability, and average bit-error probability in order to the practical system implementation which satisfies the predetermined minimum performance levels. In real time both in base stations and in mobile stations, SIR-based measurements and analyses, can be performed using specific SIR estimators as well as those for both analog and digital wireless systems (e.g., GSM, IS-54) [3-4].

Fading phenomena also remarkably affects wireless communication system performances. For mitigating fading effects and influence of co-channel interference various diversity techniques are used. Space diversity reception is an effective remedy that exploits the principle of providing the receiver with multiple faded replicas of the same information-bearing signal. Their goal is to upgrade transmission reliability without increasing transmission power and bandwidth and to increase channel capacity. By the complexity restriction put on the communication system and amount of channel state information available at the receiver, there are several principal types of combining techniques that can be performed. The best known space diversity combining techniques are MRC (Maximum Ratio Combining), EGC (Equal Gain Combining), SC (Selection Combining) and SSC (Switch and Stay Combining).

Selection combining (SC) receiver process only one of the diversity branches, and is much simpler for practical realization, comparing to othercombining techniques [5]. Other combining techniques require all or some of the amount of the channel state information of received signal and separate receiver chain for each branch of the diversity system, which increase its complexity.

In cellular systems where the level of the co-channel interference is sufficiently high as compared to thermal noise SC selection is based on the highest SIR (SIR-based selection diversity) [6].

The co-channel interference effects analysis on the wireless communication systems performance metrics has been extensively analysed [7]. Optimum combining over Rayleigh fading channels with multiple co-channel interferers was presented in [8].

Closed form expressions for cumulative distribution function (CDF) and probability distribution function (PDF) of the SC output SIR are derived, for proposed environment model, Based on this, important performance measures, the outage probability (OP) was determined. Numerical results are graphically presented in order to show the effects of the number of multiple interferers, diversity order, and the level of correlation between received desired signals and multiple interferences to the system performances. In designing a cellular mobile system, one may wish to determine optimal values of system parameters in order to achieve reasonable influence of interferers on the outage and bit-error rate occurrence.

2. SYSTEM MODEL

Let us consider the case of arbitrary number of correlated Rayleigh distributed interferers over each branch of the SC diversity system with N branches, having the same average power. This assumption is suitable for the two limiting cases that can bound the performances of any interference-limited systems, that correspond to the scenario when the interferers are on the cell edges closest to the desired user cell (worst interference case scenario) or where they are at the furthest edges (best interference case scenario) [9]. There are several wireless systems in practice that also could be adequately modelled using this assumption as explained in [10], such as single multi-antenna interferer or an interfering cluster of co-located terminals. Correlation arises between branches, when diversity system is applied on small terminals with multiple antennas. We will discuss the following case considering proposed model of constant correlation between the branches for the Nakagami-m model, given in [11]. The constant correlation model [12] can be obtained by setting $\Sigma_{i,j} \equiv 1$ for $i = j$ and $\Sigma_{i,j} \equiv \rho$ for $i \neq j$ in correlation matrices, for both desired signal and interference signal anvelopes.

Since one variable Nakagami-m (and Rayleigh, obtained from Nakagami-m as the special case for m =1) distribution could be derived from the central χ^2 (chi-square) distribution with $2L$ degrees of freedom (L independent complex Gaussian RVs) joint multivariate pdf distributions for both desired and total interfering signal correlated envelopes could be expressed by:

$$p_{R_1,\dots,R_n}(R_1,\dots,R_n) = (1-\sqrt{\rho_d})\sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty}\frac{2^n\Gamma(1+k_1+\dots+k_n)\rho_d^{\frac{k_1+\dots+k_n}{2}}}{\Gamma(1+k_1)\cdots\Gamma(1+k_n)k_1!\cdots k_n!}\left(\frac{1}{1+(n-1)\sqrt{\rho_d}}\right)^{1+k_1+\dots+k_n} \left(\frac{1}{\Omega_{d1}(1-\sqrt{\rho_d})}\right)^{1+k_1}\left(\frac{1}{\Omega_{dn}(1-\sqrt{\rho_d})}\right)^{1+k_n} R_1^{2k_1+1}\cdots R_n^{2k_n+1}\exp\left(-\frac{R_1^2}{\Omega_{d1}(1-\sqrt{\rho_d})}\right)\exp\left(-\frac{R_n^2}{\Omega_{dn}(1-\sqrt{\rho_d})}\right) \quad (1)$$

$$p_{r_1,\dots,r_n}(r_1,\dots,r_n) = \frac{(1-\sqrt{\rho_c})}{\Gamma(M)}\sum_{l_1=0}^{\infty}\cdots\sum_{l_n=0}^{\infty}\frac{2^n\Gamma(M+l_1+\dots+l_n)\rho_c^{\frac{l_1+\dots+l_n}{2}}}{\Gamma(M+l_1)\cdots\Gamma(M+l_n)l_1!\cdots l_n!}\left(\frac{1}{1+(n-1)\sqrt{\rho_c}}\right)^{M+l_1+\dots+l_n} \left(\frac{M}{\Omega_{c1}(1-\sqrt{\rho_c})}\right)^{M+l_1}\left(\frac{M}{\Omega_{cn}(1-\sqrt{\rho_c})}\right)^{M+l_n} r_1^{2M+2l_1-1}\cdots r_n^{2M+2l_n-1}\exp\left(-\frac{Mr_1^2}{\Omega_{c1}(1-\sqrt{\rho_d})}\right)\exp\left(-\frac{Mr_n^2}{\Omega_{cn}(1-\sqrt{\rho_d})}\right) \quad (2)$$

The desired signal power correlation coefficient ρ_d is defined as $cov(R_i^2,R_j^2)/(var(R_i^2)var(R_j^2))^{1/2}$, while power correlation coefficient ρ_c for interfering signal is defined as $cov(r_i^2, r_j^2)/(var(r_i^2)var(r_j^2))^{1/2}$, with R_k and r_k being the amplitudes of the desired and interference signals received at the k-th branch. $\Omega_{dk}=\overline{R_k^2}$ stands for the average desired signal power at k-th branch, while $\Omega_{ck} = \overline{r_k^2}$ is the total average interference signal power at k-th branch, where Ω_{cik}, i=1…M is the average power of the single co-channel interference, and stands $\Omega_{ck} = M\Omega_{cik}$, with M denotes the number of interferers over k-th branch.

If we define the instantaneous value of SIR at the k-th diversity branch input as $\lambda_k=R_k^2/r_k^2$. Like it was mentioned above SC chooses and outputs the branch with the largest SIR. Joint probability density function of instantaneous values of SIR at the input of multibranch SC combiner could be obtained as in [13]:

$$p_{\lambda_1,\dots,\lambda_n}(t_1,\dots,t_n) = \frac{1}{2^n\sqrt{t_1\cdots t_n}}\int_0^{\infty}\cdots\int_0^{\infty} p_{R_1,\dots,R_n}\left(r_1\sqrt{t_1},\dots,r_n\sqrt{t_n}\right)p_{r_1,\dots,r_n}(r_1,\dots,r_n)\,r_1\cdots r_n\,dr_1\cdots dr_n \quad (3)$$

And joint cumulative distribution function can be written as [13]:

$$F_{\lambda_1,\dots,\lambda_n}(t_1,\dots,t_n) = \int_0^{t_1}\cdots\int_0^{t_n} p_{\lambda_1,\dots,\lambda_n}(x_1,\dots,x_n)\,dx_1\cdots dx_n \quad (4)$$

Let $S_k = \Omega_{dk}/\Omega_{cik} = \Omega_{dk}/(\Omega_{ck}/M)$ represent the average SIR's at the k-th input branch of the multi-branch SC. Cumulative distribution function of output SIR, could be derived from (4) by equating the arguments $t_1= \dots =t_n=t$ as in [13]:

$$F_{\lambda}(t)=\underbrace{\sum_{k_1,\dots k_n=0}^{\infty}\sum_{l_1,\dots l_n=0}^{\infty}}_{2n} G_1 t^{n+k_1+\dots+k_n}\frac{\prod_{i=1}^{n} {}_2F_1\left(1+k_i,1-M-l_i;2+k_i;\frac{t}{t+\frac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}S_i}\right)}{\left(t+\frac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}S_i\right)^{1+k_i}} \tag{5}$$

with ${}_2F_1(u_1,u_2;u_3;x)$, being the Gaussian hypergeometric function [14], and:

$$G_1=\frac{(1-\sqrt{\rho_d})(1-\sqrt{\rho_c})^M\Gamma(1+k_1+\dots+k_n)\Gamma(M+l_1+\dots+l_n)\Gamma(1+M+k_1+l_1)\cdots\Gamma(1+M+k_n+l_n)}{\Gamma(M)(1+k_1)\cdots(1+k_n)\Gamma(1+k_1)\cdots\Gamma(1+k_n)\Gamma(M+l_1)\cdots\Gamma(M+l_n)k_1!\cdots k_n!l_1!\cdots l_n!}$$
$$\rho_d^{\frac{k_1+..+.k_n}{2}}\rho_c^{\frac{l_1+..+.l_n}{2}}\left(\frac{1}{1+(n-1)\sqrt{\rho_d}}\right)^{1+k_1+\dots+k_n}\left(\frac{1}{1+(n-1)\sqrt{\rho_c}}\right)^{1+l_1+\dots+l_n} \tag{6}$$

In Table 1, the number of terms to be summed in order to achieve accuracy at the desired significant digit is depicted. As we can see from the table, the values of these terms are strongly related to the correlation coefficients ρ_d and ρ_c and the number of interferers M at each branch.

Table1

Terms need to be summed in the expression for CDF of triple branch SC output SIR case to achieve accuracy at the 4th significant digit presented in the brackets

$M = 2$, $\rho_d = \rho_c = 0.2$, $S_1 = S_2$		$M = 3$, $\rho_d = \rho_c = 0.2$, $S_1 = S_2$	
S/λ = -10 dB	S/λ = 0 dB	S/λ = -10 dB	S/λ = 0 dB
15	16	17	17
19	19	22	20
22	21	24	22

$$p_{\lambda}(t)=\frac{d}{dt}F_{\lambda}(t)=\underbrace{\sum_{k_1,\dots k_n=0}^{\infty}\sum_{l_1,\dots l_n=0}^{\infty}}_{2n} G_1(1+k_1)\cdots(1+k_n)t^{n+k_1+\dots+k_n}(A_1(t)+\dots+A_n(t)) \tag{7}$$

$$A_i(t)=\left(\frac{S_i}{t+\frac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}S_i}\right)^{M+l_i}\prod_{\substack{j=1\\ j\neq i}}^{n}\frac{{}_2F_1\left(1+k_j,1-M-l_j;2+k_j;\frac{t}{t+\frac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}S_j}\right)}{(1+k_j)}$$

Fig. 1 shows the PDF of output SIR for various values of the number of multiple interferers and diversity branches.

3. OUTAGE PROBABILITY

Standard performance criterion of communication systems operating over fading channels is outage probability P_{out}. This performance measure is also used to control the noise or co-channel interference level, helping the designers of wireless communications systems to meet the QoS and grade of service (GoS) demands.

OP is usually defined as the probability that combined SNR falls below given outage threshold γ, also known as a protection ratio. Protection ratio depends on modulation technique and expected QoS. If the environment is interference limited, P_{out} is defined as the probability that the output SIR of used combiner will falls below protection ratio

$$P_{out}=P_R(\xi<\gamma)=\int_0^{\gamma}p_{\xi}(t)dt=F_{\xi}(\gamma) \tag{8}$$

Figure 2 shows the outage probability versus normalized parameter S_1/γ for balanced and unbalanced ratio of SIR at the input of the branches, various values of the number of multiple interferers and level of correlation. It can be seen from figure how the outage probability increases when the number of multiple co-channel interferers increases due to growth of interference domination. Also for this dual SC diversity case it is evident how outage probability deteriorates when higher level of correlation between the diversity branches is presented.

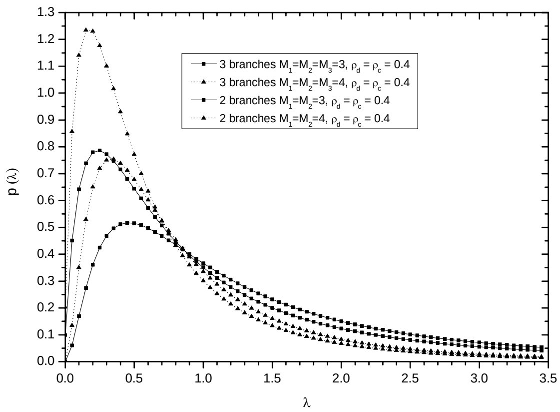

Fig. 1 – PDF of the SC output SIR can be obtained easily from previous expression

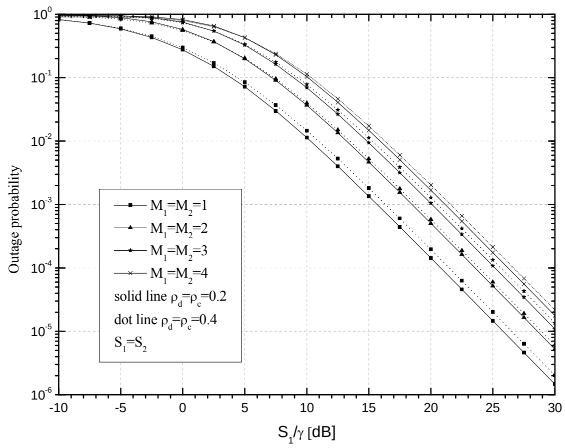

Fig. 2 – Outage probability for the two branch correlated case versus normalized parameter S_1/γ for various values of the number of multiple interferers and the level of correlation

Important system parameters, calculated based on outage probability are the reuse distance, minimum distance between any two co-channel base stations which ensures a worst case outage probability no larger than the required value, also known as CCI reduction factor and the coverage area, the area within which outage probability is guaranteed to be less than a given threshold. Capitalizing on previous evaluation and presented numerical results one may determine optimal values of system parameters for achieving reasonable level of outage in practical wireless applications.

4. CONCLUSIONS

Closed form expressions with convergence analysis for the probability density function (PDF) and cumulative distribution function (CDF) at the output of the multibranch SIR-based SC system operating over Rayleigh fading channels are derived. It is assumed that each channel experiences an arbitrary number of multiple correlated co-channel interferers. Based on them, outage probability was analysed, in order to show the effects of the number of multiple interferers, diversity order, level of correlation and input SIR unbalance on the system performances. This consideration could be taken into account, for determining optimal values of system parameters, and achieving desired influence of interferers on the outage during the design of various cellular mobile systems.

5. REFERENCES

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